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Thermal Effect of The Couple-Stress on MHD Flow in A Rotating Visco-Elastic Fluid Layer Heated from Below Saturating a Porous Medium

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ABSTRACT

This paper investigates the thermal effects of couple-stress on magnetohydrodynamic (MHD) flow in a rotating visco-elastic fluid layer saturated with a porous medium and heated from below. The investigation focuses on how couple-stress, magnetic field, rotation, and porous medium permeability influence the onset of thermal convection. A linear stability analysis is performed using the normal mode technique to derive the dispersion relation. Results indicate that couple-stress parameters and rotation tend to stabilize the system, while the magnetic field exhibits a significant damping effect on convective motion. The combined influence of these factors alters the critical conditions for instability, offering deeper insight into thermal behaviour in complex fluid systems. These results compare previous work and agree with each other.

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KEYWORDS: Magnetic Field, Rotation, Couple-stress, MHD, Thermal Convection, Visco-elastic Fluid, and Porous Medium.

INTRODUCTION

There are some important classes of fluid in technology areas, one of them visco-elastic fluid. The study of thermal convection in non-Newtonian fluids, particularly in the presence of magnetic fields and porous structures, has garnered significant attention due to its relevance in geophysical, astrophysical, and industrial applications. In recent years, the integration of couple-stress theory, visco-elastic effects, and magnetohydrodynamics (MHD) in fluid flow analysis has opened new avenues for understanding complex flow behaviours in rotating and porous environments.

In order to study of visco-elastic fluids, Oldroyd [23] introduced the general theory of viscoelastic fluid. Beard and Walters [2] investigated the viscous elastic boundary layer flow. The theory of viscous fluid flow with vertical channel, developed by Kumar et al. [8]. Sharma [25] discussed the thermal convection on visco-elastic fluid. Vivek and Pardeep [12] analyzed the dust particles on viscoelastic fluid with thermal convection. Chamkha [3] studied the problem of hydro magnetic two-phase flow in a channel. Singh and Gupta [22] investigated the thermal instability of dust particles on viscoelastic fluid flow. Chaudhary and Das [4] discussed the unsteady MHD of visco-elastic fluid flow between two parallel plates. Kumar and Mehta [11] analytic studied the visco-elastic fluid flow with hall effect. The theory of the Rayleigh instability was introduced by Wooding [29]. Siddiqui et. Al. [5] applied the Kamal Integral Transform with the Adomian Decomposition Method to solve the Riccati fractional differential equation. Their approach effectively handled nonlinearity and demonstrated rapid convergence of the analytical solution.

Pardeep and Mohan [9] investigated the hall effect of oldroydian viscoelastic fluid flow. Khanduri and Sharma [7] discussed the entropy analysis for MHD flow with thermal conductivity. Khan and Sasmal [6] studied the electro-elastic instability of visco-elastic fluid in a porous medium. Moatimid et al. [27] analyzed the EHD instability of two viscoelastic fluid flow with heat transfer. Lebon and Perez-Garcia [21] investigated the convective instability of a microfluid layer using the energy method. Their analysis provided critical conditions for the onset of convection and highlighted the stabilizing role of microstructural effects. The study offered a foundational approach to analyzed microfluidic thermal instabilities.

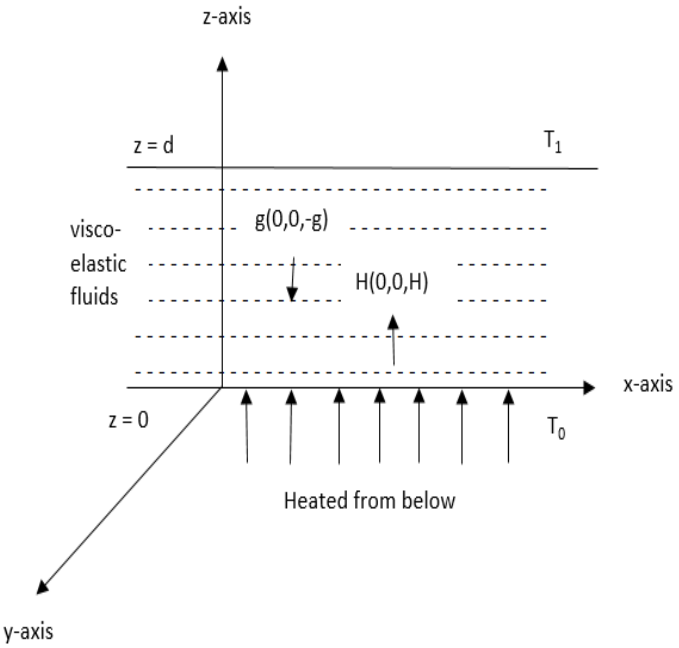
The classical theory of couple-stress fluid was developed by Stokes [28]. Kumar et al. [10] discussed the rotation on thermal instability in couple-stress viscous elastic fluid. Banyal and Singh [1] investigated the rotation on the couple-stress fluid in a porous medium. Shivakumara et al. [26] studied the onset of convection in a couple-stress fluid flow saturating a porous medium by the Galerkin method. Kumawat et al. [13–20] conducted an extensive investigation into the dynamics of micropolar and visco-elastic fluid flows under the influence of various physical effects such as couple-stress, magnetic fields, permeability, and rotation. Their studies incorporated both analytical and numerical techniques to examine thermal instability and flow behaviour in porous media.

The inclusion of suspended particles and Hall currents further enriched the modelling of real-world fluid systems. Results consistently showed that couple-stress and magnetic fields have a stabilizing influence, while permeability and particle presence modulate the onset of convection. These contributions provide significant insights into the thermal and mechanical behaviour of non-Newtonian fluids in complex environments. Xiong et al. [30] discussed the couple stress on couple stress fluid flow between parallel plates with thermal convection. Pundir [24] et al. investigated the effect of medium permeability, couple-stress parameter and magnetization on ferromagnetic fluid layer heated from below in a porous medium with hall current.

In view of the above discussion, application of this work is such as geophysics, film lubrication, chemical technology and industry. In this paper, we attempt to study the effect of medium permeability, couple-stress and magnetic field on visco-elastic fluid flow. To our knowledge, Darcy's generalized model has not yet been used to study this problem.

1. Mathematical Formulation

A horizontal, infinite, and incompressible electrically non-conducting fluid layer of thickness d is assumed, has porosity $\in$ and medium permeability k_1. The upper $z = d$ and lower limits $z = 0$ are maintained at constant but varying temperatures T_0 and T_1 such that a study adverse temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ has been continued. The magnetic field and gravity are applied along with the z-axis of the system.



The equations of continuity, motion, energy and basic's state are-

$$\nabla.\vec{q}=0 \tag{2.1}$$

$$\frac{\rho_0}{\in}\left[\frac{\partial\vec{q}}{\partial t}+\frac{1}{\in}(\vec{q}.\nabla)\vec{q}\right]=-\nabla P-\rho g\hat{e}_z+\left(\mu-\frac{\nu}{\rho_0}\nabla^2\right)\nabla^2\vec{q}$$
$$-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\vec{q}+\frac{2\rho_0}{\in}(\vec{q}\times\Omega)+\frac{\mu_e}{4\pi}(\nabla\times\vec{H})\times\vec{H} \tag{2.2}$$

$$\left[\in\rho_0 C_v+(1-\in)\rho_s C_s\right]\frac{\partial T}{\partial t}+\rho_0 C_v(\vec{q}.\nabla)T=\chi\nabla^2 T \tag{2.3}$$

$$\rho=\rho_0\left[1-\alpha(T-T_a)\right] \tag{2.4}$$

The Maxwell's equations are

$$\in\frac{\partial\vec{H}}{\partial t}=\nabla\times(\vec{q}\times\vec{H})+\in\eta\nabla^2\vec{H} \tag{2.5}$$

$$\nabla.\vec{H}=0 \tag{2.6}$$

where, P – Pressure, ρ – Fluid density, ρ_0 – Reference density, C_v- Specific heat at constant volume, $\nu-$ Couple-stress Viscosity, μ - Viscosity, μ' - Viscoelasticity, $\vec{q}$ – Velocity, $\hat{e}_z-$ Unit vector in z-direction, t – time, C_s- Specific heat of solid, ρ_s- Density of solid matrix, T – Temperature, $\chi-$ Thermal conductivity, $\alpha-$ Coefficient of thermal expansion, $\eta-$ Magnetic viscosity, T_a- Average temperature is given by $T_a=\frac{(T_0+T_1)}{2}$ and $\vec{H}=(0,0,H_z), H_z-$ Constant.

2. Basic State of the Problem

The basic state of the problem is taken as

$\vec{q}=\vec{q}_b(0,0,0),\ P=P_b(z)$ and $\rho=\rho_b(z)$

using this condition, equations (2.1) to (2.4), becomes

$$\frac{dP_b}{dz}+\rho_b g=0 \tag{3.1}$$

$$T=-\beta z+T_a=T_b(z) \tag{3.2}$$

$$\rho_b=\rho_0+\alpha\beta z\rho_0 \tag{3.3}$$

3. Linearized Perturbation Equations

Now, linearize the equation of (2.1) to (2.6), we have

$$\nabla.\vec{q}'=0 \tag{4.1}$$

$$\frac{\rho_0}{\in}\frac{\partial\vec{q}'}{\partial t}=-\nabla P'-\rho' g\hat{e}_z+\left(\mu-\frac{\nu}{\rho_0}\nabla^2\right)\nabla^2\vec{q}'-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\vec{q}'+\frac{2\rho_0}{\in}(\vec{q}'\times\Omega)+\frac{\mu_e}{4\pi}(\nabla\times\vec{h})\times\vec{H}_b$$

(4.2)

$$E\frac{\partial\theta}{\partial t}=k_T\nabla^2\theta+\beta\vec{q}.\hat{e}_z \tag{4.3}$$

$$\in \frac{\partial \vec{h}}{\partial t} = \nabla \times \left(\vec{q}\,' \times \vec{H}_b \right) + \in \eta \nabla^2 \vec{h} \tag{4.4}$$

$$\nabla . \vec{h} = 0 \tag{4.5}$$

$$\rho' = -\rho_0 \alpha \theta \tag{4.6}$$

Converting equation (4.1) to (4.6) by the following transformation $x = dx^*,\ y = dy^*,\ z = dz^*,\ \vec{q}\,' = \frac{k_T}{d}\vec{q}^*,$

$\nabla = \frac{\nabla^*}{d},\quad P' = \frac{\mu k_T}{d^2} P^*,\quad t = \frac{\rho_0 d^2}{\mu} t^*,$ and $\theta = \beta d\, \theta^*$, we have

$$\nabla . \vec{q} = 0 \tag{4.7}$$

$$\frac{1}{\in}\frac{\partial \vec{q}}{\partial t} = -\nabla P + R\theta \hat{e}_z + \left(1 - F\nabla^2\right)\nabla^2 \vec{q} - \frac{1}{K_1}\left(1 + F_1 \frac{\partial}{\partial t}\right)\vec{q} + \frac{2}{\in}\left(\vec{q} \times \Omega\right) + Q\left(\nabla \times \vec{h}\right) \times \hat{e}_z \tag{4.8}$$

$$EP_r \frac{\partial \theta}{\partial t} = \nabla^2 \theta + \vec{q} . \hat{e}_z \tag{4.9}$$

$$\in P_r \frac{\partial \vec{h}}{\partial t} = \frac{\partial \vec{q}}{\partial z} + \frac{\in P_r}{P_m} \nabla^2 \vec{h} \tag{4.10}$$

where, $R = \frac{\rho_0 \alpha \beta g d^4}{k_T \mu}$ – Rayleigh number, $F = \frac{\nu}{\rho_0 d^2},\ F_1 = \frac{\mu'}{\rho_0 d^2}$ – Viscoelastic Parameter, $P_r = \frac{\mu}{k_T \rho_0}$ – Prandtl number, $k_T = \frac{\chi}{\rho_0 C_v}$, $E = \in + \frac{(1-\in)\rho_s C_s}{\rho_0 C_v}.$ $K_1 = \frac{k_1}{d^2},$ and $W = \vec{q} . \hat{e}_z .$

4. Boundary conditions

The boundary condition is

$$W = \frac{d^2 W}{dz^2} = 0, \theta = 0 \, at \, z = 0 \, and \, z = d . \tag{5.1}$$

5. Dispersion Relation

Taking curl on both side equation (4.8), we have

$$\left[\frac{1}{\in}\frac{\partial}{\partial t} + \frac{1}{K_1}\left(1 + F_1 \frac{\partial}{\partial t}\right) - \left(1 - F\nabla^2\right)\nabla^2\right]\left(\nabla \times \vec{q}\right) = \left[R\left(\frac{\partial \theta}{\partial y}\hat{e}_x + \frac{\partial \theta}{\partial x}\hat{e}_y\right)\right] + \frac{2}{\in}\nabla \times \left(\vec{q} \times \Omega\right) + Q\frac{\partial}{\partial z}\left(\nabla \times \vec{h}\right) \tag{6.1}$$

$$Let\ \nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2},\quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},\quad D = \frac{\partial}{\partial z}.$$

Again, applying curl and z-component of equation (6.1) and (4.10), we get

$$\left[\left\{\frac{1}{\in}\frac{\partial}{\partial t} + \frac{1}{K_1}\left(1 + F_1 \frac{\partial}{\partial t}\right) - \left(1 - F\nabla^2\right)\nabla^2\right\}\right]\nabla^2 W = R\nabla_1^2 \theta - \frac{2}{\in}\Omega\left(D\zeta_z\right) + QD\left(\nabla^2 \vec{h}_z\right) \tag{6.2}$$

$$\in P_r \frac{\partial m_z}{\partial t} = D\zeta_z + \frac{\in P_r}{P_m}\nabla^2 m_z \tag{6.3}$$

Taking z-component on the both side of equations (6.1), (4.9) and (4.10), we have

$$\left[\left\{\frac{1}{\in}\frac{\partial}{\partial t} + \frac{1}{K_1}\left(1 + F_1\frac{\partial}{\partial t}\right) - \left(1 - F\nabla^2\right)\nabla^2\right\}\right]\zeta_z = \frac{2}{\in}\Omega DW + QDm_z \tag{6.4}$$

$$P_r E\frac{\partial \theta}{\partial t} = \nabla^2\theta + W \tag{6.5}$$

$$\in P_r \frac{\partial \vec{h}_z}{\partial t} = DW + \frac{\in P_r}{P_m}\nabla^2 \vec{h}_z \tag{6.6}$$

Now, the boundary condition (5.1) becomes

$W = D^2W = \theta$ at $z = 0$ to $z = 1$

Normal Mode Analysis

Let $\left[W,\ \zeta_z, B_z, \theta\right] = \left[W(z), X(z), B(z), \Theta(z)\right]\exp.\left[i k_x x + i k_y y + \sigma t\right]$

Apply normal mode of equation (6.2) to (6.6), we have

$$\left[\frac{\sigma}{\in} + \frac{1}{K_1}\left(1 + F_1\sigma\right) + F\left(D^2 - a^2\right)^2 - \left(D^2 - a^2\right)\right]\left(D^2 - a^2\right)W$$

$$= -Ra^2\Theta - \frac{2}{\in}\Omega DX + QD\left(D^2 - a^2\right)B \tag{6.7}$$

$$\left[\frac{\sigma}{\in} + \frac{1}{K_1}\left(1 + F_1\sigma\right) + F\left(D^2 - a^2\right)^2 - \left(D^2 - a^2\right)\right]X = \frac{2}{\in}\Omega DW + QDM \tag{6.8}$$

$$\left[EP_r\sigma - \left(D^2 - a^2\right)\right]\Theta = W \tag{6.9}$$

$$\left[\in P_r\sigma - \frac{\in P_r}{P_m}\left(D^2 - a^2\right)\right]M = DX \tag{6.10}$$

$$\left[\in P_r\sigma - \frac{\in P_r}{P_m}\left(D^2 - a^2\right)\right]B = DW \tag{6.11}$$

where $\sigma = \sigma_r + i\sigma_r$ is the stability parameter and $a^2 = k_x^2 + k_y^2$ is the wave number.

Now, the boundary condition becomes

$$W = D^2W = 0, \Theta = 0 \text{ at } z = 0 \text{ to } z = 1 \tag{6.12}$$

$D^{2n}W = 0$ at $z = 0$ to $z = 1,\ n > 0$.

The proper solution of equation (6.12)

$W = W_0 \sin \pi z$, W_0 - Constant.

Eliminating Θ, X and B from equations (6.7) to (6.11), putting the value of W and $b = \pi^2 + a^2$, we have

$$b\left[\frac{\sigma}{\in}+\frac{(1+F_1\sigma)}{K_1}+Fb^2+b\right]\left[\left\{\frac{\sigma}{\in}+\frac{(1+F_1\sigma)}{K_1}+Fb^2+b\right\}\left\{P_r\sigma+\frac{P_r}{P_m}b\right\}+\frac{Q\pi^2}{\in}\right]\left[EP_r\sigma+b\right]\left[P_r\sigma+\frac{P_r}{P_m}b\right]$$
$$=Ra^2\left[\left\{\frac{\sigma}{\in}+\frac{(1+F_1\sigma)}{K_1}+Fb^2+b\right\}\left\{P_r\sigma+\frac{P_r}{P_m}b\right\}+\frac{Q\pi^2}{\in}\right]\left[P_r\sigma+\frac{P_r}{P_m}b\right]-\frac{4\Omega^2\pi^2}{\in^2}\left[P_r\sigma+\frac{P_r}{P_m}b\right]^2$$
$$\left[EP_r\sigma+b\right]-\frac{Q\pi^2 b}{\in}\left[\left\{\frac{\sigma}{\in}+\frac{(1+F_1\sigma)}{K_1}+Fb^2+b\right\}\left\{P_r\sigma+\frac{P_r}{P_m}b\right\}+\frac{Q\pi^2}{\in}\right]\left[EP_r\sigma+b\right]$$

(6.13)

6. Stationary Convection

Putting $\sigma=0$ in equation (6.13), we have

$$R=\frac{b^2\left[\frac{1}{K_1}+Fb^2+b\right]+\frac{Q\pi^2 b}{\in}\frac{P_m}{P_r}}{a^2}+\frac{\frac{4\Omega^2\pi^2b^2}{a^2\in^2}\frac{P_r}{P_m}}{\left[\left\{\frac{1}{K_1}+Fb^2+b\right\}\frac{P_r}{P_m}b+\frac{Q\pi^2}{\in}\right]} \quad (7.1)$$

Neglecting the couple-stress, rotation and magnetic field (i.e. F=0 $\Omega=0$ and Q=0), we have

$$R=\frac{b^2}{a^2}\left(\frac{1}{K_1}+b\right) \quad (7.2)$$

Again, we take non-porous medium $(K_1\to\infty)$ then equation (7.2), reduce

$$R=\frac{b^3}{a^2}$$

It is derived by G. Lebon and C. Perez-Garcia.

RESULT AND DISCUSSION

Now investigate the behavior of medium permeability, rotation, couple-stress parameter, and magnetic field, we find the nature of $\frac{dR}{dK_1}$, $\frac{dR}{d\Omega}$, $\frac{dR}{dF}$, and $\frac{dR}{dQ}$ respectively from equation (7.1), then we have

$$\frac{dR}{dK_1}=\frac{-b^2}{a^2K_1^{\,2}}\left[1-\frac{\frac{4\Omega^2\pi^2b}{\in^2}\left(\frac{P_r}{P_m}\right)^2}{\left\{\left(\frac{1}{K_1}+Fb^2+b\right)\frac{P_r}{P_m}b+\frac{Q\pi^2}{\in}\right\}^2}\right] \quad (7.3)$$

$$\frac{dR}{dK_1}<0 \text{ if } Q>\frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m}$$

Thus, the medium permeability (K_1) has destabilizing effect when $Q>\frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m}$.

Neglecting the couple-stress and rotation (i.e. F=0 and $\Omega=0$), we have

$$R=-\frac{b^2}{a^2K_1^2}$$

Which is always negative, the medium permeability has destabilizing effect without any condition.
Now, from equation (7.2)

$$\frac{dR}{dF}=\frac{b^4}{a^2}\left[1-\frac{\frac{4\Omega^2\pi^2 b}{\epsilon^2}\left(\frac{P_r}{P_m}\right)^2}{\left\{\left(\frac{1}{K_1}+Fb^2+b\right)\frac{P_r}{P_m}b+\frac{Q\pi^2}{\epsilon}\right\}^2}\right] \tag{7.4}$$

$$\frac{dR}{dF}>0 \text{ if } Q>\frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m}.$$

$$Q>\frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m}$$

It is always positive if .
Thus, the couple-stress has stabilizing effect and this stabilizing effect is independent of the presence of rotation

$$\frac{dR}{d\Omega}=\frac{8\Omega\pi^2b^2}{\epsilon^2}\frac{P_r}{P_m}\frac{1}{a^2\left\{\left(\frac{1}{K_1}+Fb^2+b\right)\frac{P_r}{P_m}b+\frac{Q\pi^2}{\epsilon}\right\}} \tag{7.5}$$

It is always positive.
Thus, the rotation has stabilizing effect and this stabilizing effect is independent of the presence of couple-stress.

$$\frac{dR}{dQ}=\frac{\pi^2b^2}{a^2\epsilon}\frac{P_m}{P_r}\left[b-\frac{\frac{4\Omega^2\pi^2}{\epsilon^2}\left(\frac{P_r}{P_m}\right)}{\left\{\left(\frac{1}{K_1}+Fb^2+b\right)\frac{P_r}{P_m}b+\frac{Q\pi^2}{\epsilon}\right\}^2}\right] \tag{7.6}$$

$$\frac{dR}{dQ}>0 \text{ if } Q>\frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m}.$$

From equation (7.6), we can say that the magnetic field has stabilizing effect when the $Q>\frac{2\Omega\sqrt{b}}{\pi}\frac{P_r}{P_m}$.

7. CONCLUSIONS

- $\frac{dR}{dK_1}<0$, which implies that the medium permeability K_1 has destabilizing effect under the above condition. In the absence of couple-stress and magnetic field, the medium permeability has destabilizing effect without any condition.
- $\frac{dR}{dF}>0$, Thus the couple-stress has stabilizing effect and this stabilizing effect is independent of the presence of rotation
- $\frac{dR}{d\Omega}>0$, Thus the rotation has stabilizing effect and this stabilizing effect is independent of the presence of couple-stress.
- $\frac{dR}{dQ}>0$, We can say that the magnetic field has stabilizing effect.

According to analytically discussion, we found that the effect of medium permeability is destabilizing. In the absence of magnetic field, the effect of medium permeability is always destabilizing. The effect of rotation, magnetic field and couple stress are stabilizing. Among them the most important result that the effect of magnetic field stabilizes on the system.

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