# A Single Measure of SF 36 

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#### Abstract

Manual of SF-36 does not support computation of total score of the scale ( $S F 36_{\text {Total }}$ ) unlike multidimensional Well-being index, Human development index, etc. where total score are found for an individual or a country/region. The paper gives a method to transform scores of each item to follow normal distribution and find sub-scale scores and $S F 36_{\text {Total }}$ as convolution of normally distributed item scores, parameters of which can be estimated from the data. Addition and arithmetic aggregation of item scores is meaningful due to normality of such scores following same distribution with different parameters. Normally distributed $S F 36_{\text {Total }}$ scores avoid limitations of summative Likert scores and facilitate parametric statistical analysis including statistical tests of equality of average of $S F 36_{\text {Total }}$ in two groups for cross-sectional as well as longitudinal data. In addition, $S F 36_{\text {Total }}$ helps to find responsiveness of SF-36 by assessing changes in two time periods either for an individual or a sample of individuals which in turn helps to draw progress-paths. Normality of proposed scores also helps to find psychometric features like factorial validity, discriminating value and reliability. The proposed method is illustrated with hypothetical data.


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## Introduction:

SF-36 is the short form of the Health Survey questionnaire (SF-36). With 36 items distributed over eight sub-scales, the SF-36 is a self-reported, patientreported questionnaire and is a popular tool to evaluate health-related quality of life (HRQoL). Items in SF 36 have different formats: 28 items in Likert format; seven items are binary, and another item relates to health transitions over the previous year. Raw item scores are rescaled to ranges between 0 and 100, and a high
score indicates a more favourable health state. This requires reverse scoring for negatively worded items to have a uniform direction of scores. A zero score on an item is problematic as it tends to reduce the values of mean, variance, covariance, and correlation with that item. Analysis like expected values (score multiplied by the corresponding probability) is not meaningful when zero is attached to one level of an item. The sub-scale wise distribution of items in SF-36 is shown in Table 1.

Table 1: Sub-sales of SF 36, corresponding items and Recorded values

| values |  |  |
| :---: | :---: | :---: |
| Sub-scales | Total no. of items \& scale points | Item numbers \& Recorded values of Response choices |
| Physical functioning | 10 (3points) | $3,4,5,6,7,8,9,10,11,12$ (choice of 1 is recorded as 0 and choice of 3 is recorded as 100) |
| Role limitations due to physical health | 4 (Yes No type) | $13,14,15,16$ (choice of 1 is recorded as 0 and choice of 2 is recorded as 100) |
| Role limitations due to emotional problems | 3 (Yes No type) | 17, 18,19 (choice of 1 is recorded as 0 and choice of 2 is recorded as 100) |
| Energy/ Fatigue | 3 (6-point) | 23,27 (choice of 1 is recorded as 100 and choice of 6 is recorded as 0 ); 2931 (choice of 6 is recorded as 100 and choice of 1 is recorded as 0 ) |
| Emotional wellbeing | 6 (6-points) | $24,25,26,28,30$ (choice of 6 is recorded as 100 and choice of 1 is recorded as 0 ) and 26 (choice of 1 is recorded as 100 and choice of 6 is recorded as 0 ) |
| Social functioning | 2 (5-point) | 20 (choice of 1 is recorded as 100 and choice of 5 is recorded as 0 ) and 32 (choice of 1 is recorded as 0 and choice of 5 is recorded as 100) |
| Pain | Item 21(6point) \& Item 22 (5point) | 21 (choice of 1 is recorded as 100 and choice of 6 is recorded as 0 ) and 22 (choice of 1 is recorded as 100 and choice of 5 is recorded as 0) |
| General health | 5 (5-point) | $1,34,36$ choice of 1 is recorded as 100 and choice of 5 is recorded as 0 ) and 33,35 choice of 1 is recorded as 0 and choice of 5 is recorded as 100) |

Since the total score of an individual by a single number () is not supported by the Manual of SF-36 (http://www.webcitation.org/6cfeefPkf), due to several dimensions being measured by the scale, descriptive statistics and psychometric qualities like reliability, validity, etc. are reported separately for each of the eight sub-scales. Different item formats and different numbers of items in sib-classes result in different values of the mean, standard deviation (SD), distribution of item scores, and reliability, validity, and discriminating power of the SF-36.
Studies to investigate the factor structure of the SF-36 using factor analysis (Guermazi et al. 2012) and structural equation model analysis (Anagnostopoulos et al. 2005) confirmed the multidimensional structure of the SF-36. However, there are measures of variables having a multidimensional nature, like the well-being index, the human development index, etc., where a total score is found for an individual or a country or region.
Physical Component Summary (PCS) and Mental Component Summary (MCS) are the two distinct concepts measured by SF-36. Calculation of PCS and MCS requires the use of special algorithms, which are controlled by a private company (http://www.webcitation.org/6cfdiZOJI).

Lins \& Carvalho (2016) found a number of articles suggesting calculation of the total score of SF-36 in different ways, including the arithmetic average of eight sub-classes (Arefnasab et al. 2013; Boccard et al. 2014; Boi et al. 2012) or MCS and PCS (Barnett et al. 2013; Pekmezovic et al. 2015). One major problem with summative item scores is the assumption of equidistance between response categories, despite the fact that additions of ordered categorical data are not strictly appropriate for ordinal data (Svensson, 2001). Thus, the arithmetic average would be meaningful if each item's scores were transformed into equidistant scores following a similar distribution. Available norms based on scores of SF-36 for various populations in several countries, including sub-populations by age and gender, can be compared with the general population (Ware et al., 2005; Australian Bureau of Statistics, 1997). However, this requires a sound scoring system for each sub-scale of the SF-36 questionnaire and a method of aggregation of item scores to sub-scale scores and sub-scale scores to get the questionnaire or scale score for each individual.
The paper aims at transferring raw item scores of different sub-scales of SF 36 to follow normal distribution in a desired score range and finding the total score of SF-36 scores as the sum of normally distributed item scores or, equivalently, sub-class scores obtained as the sum of scores of relevant items of a sub-class. Such transformed scores satisfy desired properties and facilitate the assessment of changes in scale scores over time, either for an individual or a sample of individuals, and the drawing of progress paths.
After a literature survey highlighting the limitations of existing methods, a proposed method is introduced and its associated properties are elaborated. This is followed by empirical illustrations and rounded up by discussing salient outcomes and recommendations.

## Literature survey:

Grassi et al. (2007) transferred scores of Likert items on the SF-36 to binary formats using Multiple Correspondence Analysis (MCA). However, MCA does not provide a unique way to transform. Questions can be raised on why to transform Likert scores to binary scores, which amounts to a restriction of data fluctuations due to the reduction of response categories. Why not transform the original scores of each item into a K -point scale where $\mathrm{K}=2,3,4,5$, or 6 ? All such questions can be avoided if item scores are transformed to follow a similar distribution, say "normal. Taft et al. (2001) investigated the relative contribution of the subscales to PCS or MCS using factor scoring coefficients derived from PCA and found an inverse relationship between PCS and MCS, implying that good physical health presupposes poor mental health and vice versa. However, assumptions of PCA like at least interval-level measurements (i.e., equidistant), linearity between observed variables, bivariate normal distribution for each pair of observed variables, etc. are
not satisfied by ordinal item scores emerging from SF36.

Reporting of the analysis of SF-36 data starts with descriptive statistics showing the mean, SD, etc. for each sub-class. But addition is not meaningful for ordinal data since Likert scores fail to satisfy the equidistant property. Thus, $X>Y$ oXY isis meaningless since the arithmetic mean is not defined for ordinal scales (Hand, 1996). Non-admissibility of meaningful addition implies SD, coefficient of variation (CV), correlation, Cronbach, may not be meaningful, and analysis like regression, PCA, FA, SEM, etc. may give distorted results. In addition, if the assumptions of the techniques used to analyse the data are not satisfied, the results may go wrong. For example, a high correlation (between two variables $X$ and $Y$ ) is taken as a linear relationship between $X$ and $Y$. However, linearity implies high correlation, but the converse is not true. For example,if $X$ takes integer values from 1 to 30, it is 0.97; tatistics showing the mean, SD, etc. for each sub-class. But addition is not meaningful for ordinal data since Likert scores fail to satisfy the equidistant property. Thus, $X>Y$ or $X Y$ is meaningless since the arithmetic mean is not defined for ordinal scales (Hand, 1996). Non-admissibility of meaningful addition implies SD, coefficient of variation (CV), correlation, Cronbach, may not be meaningful, and analysis like regression, PCA, FA, SEM, etc. may give distorted results. In addition, if the assumptions of the techniques used to analyse the data are not satisfied, the results may go wrong. For example, a high correlation (between two variables $X$ and $Y$ ) is taken as a linear relationship between $X$ and $Y$. However, linearity implies high correlation, but the converse is not true. For example, if $X$ takes integer values from 1 to 30 , it is $0.97 ; 0.92$, despite each being non-linearly related to $X$. To fit the regression line of $Y$ on $X$ (or $X$ on $Y$ ), it is necessary to test the hypothesis where n denotes the number of observations and the test of homoscedasticity, reflecting that the residuals are equally distributed. Hawkin's test is a test of homoscedasticity as well as multivariate normal. Here, the error score for OR did not follow the normal distribution, indicating a violation of the assumption of OLS. This is an example to show how violations of assumptions in statistical analysis may mislead the results.

## Major shortcomings of summative Likert scores are:

Addition is not meaningful with ordinal Likert scores (Jamieson, 2004). Do not satisfy equidistant property (Bastien and Morin, 2001). Item levels may mean different things to different subjects responding to the scale (Kampen and Swyngedouw, 2000). Items with varying contributions to total scores, different reliabilities as item-total correlations, different factor loadings, etc. do not justify assigning equal importance to them (Parkin et al. 2010). This often results in tied scores as different individuals may get the same scale score based on different patterns of responses to the items.

Thus, a sub-scale cannot discriminate between individuals with the same score.
Unknown and different distributions of it on the on score. A score of 50 in sc with aiumber 5 -average but the same score in scale $Y$ with 10 items, each in 5fformat is the maximum possible score. Interpretation and further operations of sum of X and Y are problematic when X and Y follow different unknown distributions.
A questionnaire may have several scales (a battery of Likert scales) where scale length (number of items) and item formats (number of levels) vary. Here, joint distribution of scale scores is problematic without knowledge of scale distributions.
The distributions of scores emerging from Likert scales are skewed and do not follow a normal distribution. Normality is the common assumption of statistical techniques such as PCA, AVOVA, goodness of fit of regression equations, estimation, testing, etc. (Montgomery and Runger 2006).
The need to consider response a category along with the format of the questionnaire was suggested (Khadka et al. 2012). An increase in the number of response categories usually increases Cronbach's alpha and factorial validity (Lozano et al. 2008). Su et al. (2014) found that WHOQOL-BREF scores were more reliable than the SF-36 scores for assessing people with schizophrenia. The reliability of scales is samplespecific. For normally distributed scores, it is possible to have a population estimate of scale variance and also variance for each item and obtain a population estimate of Cronbach alpha. The generic scale SF-36 has been applied to various disease groups. Empirically, studies showed adequate internal consistency and reliability in terms of Cronbach's for most of the sub-scales except social functioning and general health. For example, alpha for social functioning in patients with brain tumours was 0.53 (Bunevicius, 2017) and 0.45 in patients with coronary artery disease (Alonso, 1995). Cronbach's alpha works best if the scale has one dimension and satisfies the assumptions of uncorrelated errors, tau-equivalence, and normality (Sijtsma and van der Ark, 2015). No attempt was found to report the reliability of the SF-36 scale as a battery consisting of eight sub-scales.
For longitudinal data, Busija et al. (2008) considered that a sub-scale of the SF-36 may have floor or ceiling effects if respondents reported the worst (0) or best (100) possible scores of at least $15 \%$. Here, $15 \%$ is arbitrary. It could be better to find the distribution of SF36 scores, convert it to a symmetrical distribution like the normal distribution, and consider respondents lying outside
Changes in scores at different time periods could be either due to a real change in health status, the effects of random error, or both. Sensitivity of sub-scales using Minimal Detectable Change (MDC) by de Vet et al. (2001) requires partitioning of the within-person variations as between-assessment variance plus the residual variance and computation of the standard error
of measurement (SEM) as the square root of this residual within-person variance (Masse et al. 1997).
MDC for a group obtained as standard errors of the sample means is influenced also by sample size. An individual is categorized under deteriorated if amount of decrease of his/her score exceeds the MDC at individual level. Purpose of MDC is to assess changes which exceed the measurement error by 1.96* SEM presumes normality (Ware et al.2005).Busija et al. (2008) concluded against use of SF-36 due to low sensitivity of SF-36 subscales (except General Health across the intervention groups) for patients undergoing orthopedic surgery. Ware et al, (2005) attempted to compare the published norms for SF-36 over age and gender. However, this may be problematic, in case of small value of the standard errors of published norm scores.

## Proposed method:

Pre-adjustment: Ensure that response categories for each item are ordered from low to high, i.e., the lowest level is marked as 1 , the second lowest level is marked as 2 , and so on. This requires reverse scoring of each "negatively phrased" item.
As per the method given by Chakrabartty (2021), raw discrete item scores ( X ) are transformed to continuous, equidistant sores ( E ), which are standardised to follow and further transformed by linear transformation to follow Normal in the desired score range [ 0,100 ].
Equidistant scores:
Let's denote the raw score of a respondent in the i-th item if he or she chooses the $j$-th response category. If the item is 5 -point, weighted score $(W S)=$ where are different for different levels of the i-th item satisfying Scores of the i-th item will be equidistant and monotonic if,, and form an arithmetic progression (AP) with a common difference (CD)>0.
Forthe i-th item, find maximum ( $f_{i \max }$ ) and minimum frequency $\left(f_{i \text { min }}\right)$ of the levels. Find initial weights $\omega_{i j}=$ $\frac{f_{i j}}{n}$. Arrange the $\omega_{i j}^{\prime} s$ so that $\omega_{i 1}<\omega_{i 2}<\omega_{i 3}<$ $\omega_{i 4}<\omega_{i 5}$ where $\omega_{i 1}=\frac{f_{i \min }}{n}$ and $\omega_{i 5}=\frac{f_{i \max }}{n}$. Let intermediate weight $W_{i 1}=\omega_{i 1}$
The common difference $\alpha$ can be found as $\alpha=$ $\frac{5 f_{i \text { imax }}-f_{i \text { min }}}{4 n}$ since $W_{i 1}+4 \alpha=5 \mathrm{~W}_{\mathrm{i} 5}$
Define other intermediate weights as $W_{i 2}=\frac{\omega_{i 1}+\alpha}{2}, W_{i 3}=$ $\frac{\omega_{i 1}+2 \alpha}{3} ; W_{i 4}=\frac{\omega_{i 1}+3 \alpha}{4}$; and
$W_{i 5}=\frac{\omega_{i 1}+4 \alpha}{5}$. Get final weights $W_{i j(\text { Final })}=\frac{W_{i j}}{\sum_{j=1}^{5} W_{j}}$ enabling $\sum W_{i j(\text { Final })}=1$ and
$j . W_{j(\text { Final })}-(j-1) \cdot W_{(j-1)(\text { Final })}=$ constant
However, value of constant will be different for different items, when the process is repeated for each item Observations:
i) $W_{j(\text { Final })}$ are based on empirical probabilities.
ii) $f_{i j}=0$ is the zero value of the transformed scores.
iii) Generated scores (E) as weighted sum are equidistant and continuous.
iv) The method can be used for items with different number of response-categories including binary items. Transform E-scores of the i-th item byZ $\mathrm{i}_{i j}=$ $\frac{x_{i j}-\overline{X_{l}}}{\operatorname{SD}\left(X_{i}\right)} \sim N(0,1)$.
Take linear transformation of Z -scores to P -scores by:

$$
P=(99) *\left[\frac{\left(z_{i j}-\operatorname{Min}\left(z_{i j}\right)\right.}{\operatorname{Max}\left(z_{i j}\right)-\operatorname{Min}\left(z_{i j}\right)}\right]+1
$$

(1)

For the i-th item, $P_{i} \sim N\left(\mu_{i}, \sigma_{i}^{2}\right)$ and $1 \leq P_{i} \leq 100$ where estimates of $\mu_{i} \mathrm{and} \sigma_{i}^{2}$ are obtained from the data. Pscore of an item as per equation (1) can be obtained irrespective of length of scale and width of items.
Sub-class score of an individual is taken as sum of normally distributed P -score of relevant items which will follow normal with mean $\sum_{i} \mu_{i}$ and SD $=\sqrt{\sum \sigma_{i}^{2}+2 \sum_{i \neq j} \operatorname{Cov}\left(P_{i}, P_{j}\right)}$. Scale/battery score or total SF-36 score $\left(S F 36_{\text {Total }}\right)$ is similarly taken as sum of sub-class scores, which also follows normal.
Properties
Sub-class scores $\left(D_{i}\right)$ and scalescores $\left(S_{i}\right)$ of the i-th individual follow normal distribution.
Normality ensures meaningful computation of arithmetic average, SD, correlation, etc. and facilitates statistical analysis under parametric set up including unbiased estimates of population mean ( $\mu$ ), population variance ( $\sigma^{2}$ ), confidence interval of $\mu$, and testing of null hypothesis like $H_{0}: \mu_{1}=\mu_{2}$ or $H_{0}: \sigma_{1}^{2}=\sigma_{2}^{2}$ etc. across time and space.
2. Progress registered by the i-th person in two successive time-periods can be quantified in percentage by by $\frac{\text { SF36 Total } l_{i(t)} \text {-SF36 Total }}{i(t-1)}$ SF36 Total ${ }_{i(t-1)} 100$ which also quantifies responsiveness of the scale and effectiveness of a treatment plan. Deterioration is indicated when
SF36Total $_{(t)}-$ SF36 Total $_{(t-1)}>0$ implying progress in t -th period over ( $\mathrm{t}-1$ )-th period. The reverse is true for $S_{F} 3$ Total $_{i(t)}<$ SF36 Total $_{i(t-1)}$. Deterioration in terms of $S F 36_{\text {Total }}$ scores may be probed to find extent of deterioration in sub-class scores for possible corrective actions. Similarly, progress for a group of persons is reflected if $\overline{S F 36 \text { Total }_{l(t)}}>\overline{\text { SF36Total }_{l(t)}}$. Denoting $S F 36_{\text {Total }}$ Scores as S, one can test $H_{0}: \mu_{S_{t}}=\mu_{(t-1)}$
3. The graph of progress and/or deterioration of a patient or sample of patients at various time points can be used to compare pattern of progress or HRQoL of patient(s) from the starting year

## Benefits:

In addition, the normality of the proposed method can also help to find psychometric properties of
SF-36 in better fashion.
Normally distributed scores satisfy the assumptions of PCA and enable factorial validity.
in terms of the ratio of the first eigenvalue to the sum of all eigenvalues, i.e., validity $=$, where is the first principal component with the highest eigenvalue
reflecting the main factor for which the scale was developed. Note that validity accounts for a percentage of overall variability. Such factorial validity avoids the problems of construct validity and the selection of criterion scales (Parkerson et al. 2013).
The generic scale SF-36 resulted in various values of Cronbach's alpha for different groups. Population estimates of the variance of each item and scale are possible for normally distributed sub-class scores. Such estimates can be used to find the population estimate of Cronbach's alphafor a subclass as
$\hat{\alpha}=\left(\frac{n}{n-1}\right)$
$\left.\frac{\text { Sum of estimates of variance of items in the sub-class) }}{\text { Estimate of variance of the sub-class }}\right)$
(2)

Reliability of the SF-36 as a battery consisting of eight sub-scales as a function of reliabilities of the subscales can be obtained as follows:
wherer $r_{t t(i)}$ and $S_{x i}$ denote respectively estimate of reliability and SD of the $i$-th sub-class.
Discriminating value of a QOL scale is poorly defined or not defined. Mere observation that average QOL score for a group of healthy adults was higher than the group of patients suffering from chronic illnesses, like cancer, etc. may not be sufficient to conclude that the scale has good discriminating value. Such value needs to be quantified. Discriminating value reflects ability of the scale to distinguish between individuals that have different degrees of the underlying construct (e.g. more or less severe disease). Discriminating value of Likert item $\left(\right.$ Disc $\left._{i}\right)$ and test ( Disc $_{\text {Test }}$ )can be computed by Coefficient of variation (CV) where $\operatorname{Disc}_{i}=\frac{S D_{i}}{\text { mean }_{i}}$ and Disc $c_{\text {Test }}=\frac{S D_{\text {Test }}}{\text { Mean }_{\text {Test }}}$. Relationship between Cronbach $\alpha$ and $\operatorname{Disc}_{\text {Test }}$ (with m -items) was derived as

$$
\begin{equation*}
\alpha=\left(\frac{m}{m-1}\right)\left(1-\frac{\sum_{i=1}^{m} \bar{X}_{l}^{2} \cdot . D i s c_{i}^{2}}{\bar{X}^{2} \cdot D i s c_{T}^{2}}\right) \tag{4}
\end{equation*}
$$

Since, variance of the i-th item $S_{X_{i}}^{2}=\bar{X}_{l}^{2} . D i s c_{i}^{2} \forall \mathrm{i}=1,2$, $\ldots, \mathrm{m} \Rightarrow \sum_{i=1}^{m} S_{X_{i}}^{2}=\sum_{i=1}^{m} \bar{X}_{l}^{2} . D i s c_{i}^{2}$ and Test variance $S_{X}^{2}=\bar{X}^{2}$. Disc $_{T}{ }^{2}$
It can be proved that $\left(\text { Disc }_{\text {Test }}\right)^{2}=\frac{C V_{\text {Truescores }}{ }^{2}}{r_{\text {tt }}}$ where $r_{t t}=\frac{s_{T}^{2}}{s_{X}^{2}}$
Thus, test reliability and Disc $_{\text {Test }}$ are related by a negative non-linear relationship.
A classification of individuals to a finite number of mutually exclusive categories needs to decide boundary points ensuring that members within a class/cluster are similar (small within group variance) and members between classes/clusters are dissimilar (high between group variance). Efficiency of classification needs to be evaluated. Quartile clustering helps in classification of a
group of individuals in four mutually exclusive classes viz. the quartiles $Q_{1}, Q_{2}, Q_{3}, Q_{4}$ (Goswami and Charabarti, 2012). Quartile clustering of proposed scale scores of SF 36 following normal distribution may be adopted because it is simple, appealing, adds clear meaning to the clusters, and gives equal probability to each quartile i.e. $\int_{0}^{Q_{1}} f(x) d x=\int_{Q_{1}}^{Q_{2}} f(x) d x=$ $\int_{Q_{2}}^{Q_{3}} f(x) d x=\int_{Q_{3}}^{Q_{4}} f(x) d x$

## Empirical illustration:

Hypothetical data involving 100 individuals in each subclass of the SF-36 was considered for illustration of the proposed method.

## Equidistant scores:

Different weights were assigned to different response categories of different items to get an equidistant score. An example of the computation of weights to get equidistant scores for general health with five items, each in 5-point form (sub-scale 8), is given in Table 2.

Table 2: Different Weights to response-categories of five Items of

| Item | Weights to different response categories |  |  |  |  | Common differenc e (CD) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | RC 1 | RC 2 | RC 3 | RC 4 | RC 5 |  |
| 1 | $\begin{aligned} & 0.02 \\ & 5121 \end{aligned}$ | $\begin{aligned} & 0.1860 \\ & 53 \end{aligned}$ | $\begin{aligned} & 0.2396 \\ & 96 \end{aligned}$ | $\begin{aligned} & 0.2665 \\ & 18 \end{aligned}$ | $\begin{aligned} & 0.282 \\ & 612 \end{aligned}$ | 0.346984 |
| 2 | $\begin{aligned} & 0.05 \\ & 0361 \end{aligned}$ | $\begin{aligned} & 0.1880 \\ & 66 \end{aligned}$ | $\begin{aligned} & 0.2339 \\ & 67 \end{aligned}$ | $\begin{aligned} & 0.2569 \\ & 18 \end{aligned}$ | $\begin{aligned} & 0.270 \\ & 689 \end{aligned}$ | 0.32577 |
| 3 | $\begin{aligned} & 0.08 \\ & 2432 \end{aligned}$ | $\begin{aligned} & 0.1906 \\ & 23 \end{aligned}$ | $\begin{aligned} & 0.2266 \\ & 87 \end{aligned}$ | $\begin{aligned} & 0.2447 \\ & 19 \end{aligned}$ | $\begin{aligned} & 0.255 \\ & 538 \end{aligned}$ | 0.298815 |
| 4 | $\begin{aligned} & 0.03 \\ & 319 \end{aligned}$ | $\begin{aligned} & 0.1866 \\ & 96 \end{aligned}$ | $\begin{aligned} & 0.2378 \\ & 65 \end{aligned}$ | $\begin{aligned} & 0.2634 \\ & 49 \end{aligned}$ | $\begin{aligned} & 0.278 \\ & 8 \end{aligned}$ | 0.340202 |
| 5 | $\begin{aligned} & 0.07 \\ & 863 \end{aligned}$ | $\begin{aligned} & 0.1903 \\ & 2 \end{aligned}$ | $\begin{aligned} & 0.2275 \\ & 5 \end{aligned}$ | $\begin{aligned} & 0.2461 \\ & 65 \end{aligned}$ | $\begin{aligned} & 0.257 \\ & 334 \end{aligned}$ | 0.30201 |

Legend: RC- j denotes the j -th Response-category $\forall$ $j=1,2,3,4,5$

$$
\mathrm{CD}=j W_{j}-(j-1) W_{j-1} \forall j=2,3,4,5
$$

## Observations:

E -score of an item is obtained as a weighted sum, i.e. $\sum_{i=1}^{5} i . W_{i(\text { Final })}$ is continuous, monotonic and equidistant since $5 W_{5(\text { Final }}-4 W_{4(\text { Final })}=4 W_{4(\text { Final })}-$ $3 W_{3(\text { Final })} \quad=3 W_{3(\text { Final })}-2 W_{2(\text { Final })}=2 W_{2(\text { Final }}-$ $W_{1 \text { (Final) }}>0$. However, values of CD were different for different items. E-score of
A sub-class was the sum of item-wise E-scores.

## Observed Score range of sub-classes:

E -score of sub-classes reduced the range of scores. Score range of $P$-score in $[1,100]$ increased the range of scores. Score rangeof sub-classes under X, E, and Pare shown in Table 3

Table 3: Observed range of scores for each sub-scale

| Sub-classitems and | Raw score(X) |  | Equidistant score <br> (E) |  | P-score in [1,100] following normal |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Max | Min | Max | Min | Max | Min |
| Physical functioning (10 items, 3-points) | 28 | 11 | 10.14867 | 3.099054 | 859.5508 | 63.82017 |
| Physical Role functions (4 binary items) | 8 | 4 | 4.22 | 1.86 | 400.00 | 4.000008 |
| Emotional Role functions (3 binary items) | 6 | 3 | 3.167311 | 1.584366 | 235.3833 | 3.000163 |
| Energy/ Fatigue (4 items, 6 -point) | 22 | 8 | 4.716457 | 1.149574 | 360.40 | 79.19447 |
| Emotional wellbeing (5 items, 6-points) | 27 | 7 | 5.382082 | 0.87412 | 440.3357 | 44.60047 |
| Social Functioning (2items, 5point) | 9 | 2 | 2.7665 | 0.075482 | 200.00 | 1.999831 |
| Pain(Item 21, 6-point \& Item 22, 5-point) | 11 | 2 | 2.506782 | 0.154319 | 200.00 | 2.000081 |
| General Health (5 items, 5-point) | 22 | 11 | 5.771311 | 2.116792 | 425.75 | 151.249 |

## Tied scores:

Raw scores resulted in number of tied scores unlike E-scores and Z-scores. For example, seven persons were tied at a raw score of 13 in the 8 th sub-scale. The tie was broken by E-score and Z-score. Details areshown in Table 4.

Table 4: Illustrative tied raw score and corresponding E-score and Z-score

| Sl. | Raw score of items |  |  |  |  |  |  | Total score under |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. | Item 1 | Item 2 | Item 3 | Item 4 | Item 5 | X | E |  |  |  |
| 1 | 2 | 5 | 1 | 4 | 1 | 13 | 2.940406 | -2.74749 |  |  |
| 2 | 5 | 1 | 1 | 4 | 2 | 13 | 2.980287 | -2.50717 |  |  |
| 3 | 3 | 1 | 2 | 2 | 5 | 13 | 2.810761 | -1.58281 |  |  |
| 4 | 4 | 1 | 2 | 1 | 5 | 13 | 2.817543 | -2.94016 |  |  |
| 5 | 4 | 1 | 2 | 2 | 4 | 13 | 3.202719 | -1.99881 |  |  |
| 6 | 3 | 1 | 1 | 3 | 5 | 13 | 2.852148 | -2.83989 |  |  |
| 7 | 4 | 1 | 4 | 3 | 1 | 13 | 2.887535 | -2.7765 |  |  |

## Observations:

- If i and j aretwo different persons, $E_{i} \neq E_{j}$ and $Z_{i} \neq Z_{j}$ even if $X_{i}=X_{j}=13$ in a sub-class
- E-scores and Z-scores consider the pattern of responses to get a particular score and each helps to assign unique ranks to the subjects responding to SF-36.
- Variance in terms of $X$-score for the set of subjects with tied score as 13 was zero and thus, the scale failed to discriminate them. $-\bar{E} \neq \bar{Z} \neq \bar{X} ; \operatorname{SD}(E)=0.136438$ and $S D(Z)=0.505262$ for the 7 -persons with $X=13$


## Descriptive statistics:

Descriptive statistics for sub-classes and $S F 36_{\text {Total }}$ are shown in Table 5.
Table 5: Mean, SD, CV and correlation ( $r_{X P}$ ) of sub-class scores and total SF 36 scores

| Sub-class | Raw scores (X) |  |  | Normally distributed P-scores |  |  | $\boldsymbol{r}_{\boldsymbol{X P}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Mean | SD | CV | Mean | SD | CV |  |
| Sub-class 1 | 20.29 | 3.677106 | 0.181228 | 488.3476 | 171.037 | 0.350238 | 0.99609 |
| Sub-class 2 | 6.11 | 0.815197 | 0.13342 | 312.4843 | 79.40792 | 0.254118 | 0.52294 |
| Sub-class 3 | 4.32 | 0.723069 | 0.121025 | 162.1078 | 64.40614 | 0.397304 | 0.89897 |
| Sub-class 4 | 16.66 | 2.999057 | 0.180015 | 254.502 | 60.05119 | 0.235956 | 0.99910 |
| Sub-class 5 | 18.41 | 3.621192 | 0.196697 | 269.2216 | 71.84771 | 0.266872 | 0.99864 |
| Sub-class 6 | 6.18 | 1.629154 | 0.263617 | 106.9399 | 41.04252 | 0.38379 | 0.93273 |
| Sub-class 7 | 7.29 | 2.066056 | 0.28341 | 93.52947 | 38.24569 | 0.408916 | 0.57372 |
| Sub-class 8 | 16.39 | 2.870593 | 0.175143 | 291.7759 | 70.86596 | 0.242878 | 0.98828 |
| SF36 Total | 95.57 | 8.932129 | 0.093462 | 1666.325 | 267.0231 | 0.160247 | 0.89856 |
|  |  |  |  |  |  |  |  |

## Observations:

Lower value of CV reflects more score consistency. Here,lowest CVwas registered by the sub-class 4 for Pscore and sub-class 8 for X -scores,
Theoretically defined test reliability $r_{t t}=\frac{s_{T}{ }^{2}}{s_{X^{2}}}=\frac{s_{T}{ }^{2} / \bar{T}^{2}}{s_{X} / 2 \bar{X}^{2}}=$ $\frac{C V_{T}^{2}}{C V_{X}^{2}} \Rightarrow C V_{X}^{2}=\frac{C V_{T}^{2}}{r_{t t}} \quad$ where $C V_{X}$ denotes CV for observed High correlation between X and P for each sub-class except sub-class 2 and 7 (moderate correlations) indicate not much disturbance of data structure due to the transformations. Correlation between X and P of

SF36 Total scores was 0.898 . However, poor admissibility of X scores might have distorted the results and extent of distortions is not known.
scores and similarly, $C V_{T}$ stands for CV for true scores. Thus, there is a negative relationship between $C V_{X}^{2}$ and $r_{t t}$ (as per theoretical definition).

## Correlations:

Inter-subscale correlations and subscale-battery correlations for P -scores are shown in Table 6.

Table 6: Correlation matrix between Sub-classes and Battery

|  | SC 1 | SC 2 | SC 3 | SC 4 | SC 5 | SC 6 | SC 7 | SC 8 | Battery |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| SC 1 | 1 | 0.0486 | 0.0223 | 0.1201 | 0.1602 | 0.0507 | -0.0201 | -0.0248 | 0.7234 <br> $(0.4944)$ |
| SC 2 |  | 1 | -0.1494 | -0.0788 | -0.0607 | 0.0277 | 0.0210 | -0.0769 | 0.1339 <br> $(0.0191)$ |
| SC 3 |  |  | 1 | -0.0159 | -0.0037 | 0.0345 | -0.0347 | -0.0002 | 0.2234 <br> $(0.0606)$ |
| SC 4 |  |  |  | 1 | 0.7005 | 0.1937 | 0.0807 | 0.0319 | 0.5216 <br> $(0.6985)$ |
| SC 5 |  |  |  |  | 1 | 0.1540 | 0.0646 | -0.0699 | 0.5314 <br> $(0.7069)$ |
| SC 6 |  |  |  |  |  | 1 | 0.0121 | 0.5988 | 0.4453 <br> $(0.4624)$ |
| SC 7 |  |  |  |  |  |  | 1 | 0.2897 | 0.2402 <br> $(0.2072)$ |
| SC 8 |  |  |  |  |  |  |  | 1 | 0.3571 <br> $(0.4179)$ |

Legend: Figures within brackets represent correlations in terms of X -scores SC - i : i-th sub-class, $\mathrm{i}=1,2, \ldots$

## Observations:

Correlations between sub-classes were mostly poor with the exceptions of 4th and 5th subclasses and also 6th and 8th subclasses. Such poor correlations tend to indicate more than one factor from PCA or FA.
Subclass reliability in terms of correlation with battery scores (in line with item-total correlations) ranged between 0.1339 to 0.7234 for P-scores and between 0.0191 to 0.7069 for X -scores.

Distributions:
LetP-scores of i-th sub-class be denoted by $P_{i}$. Parameters of normally distributed $P_{i}$ and $S F 36_{\text {Total }}$ are shown in Table 7.

Table 7: Parameters of normally distributed Subscale score and SF36 Total $^{\text {score }}$

| Score | Normal distribution with parameters |  |
| :---: | :--- | :--- |
|  | Mean | SD |
| $P_{1}$ | 488.3476 | 171.0377 |
| $P_{2}$ | 53.47 | 49.65976 |
| $P_{3}$ | 109.7719 | 64.40614 |
| $P_{4}$ | 253.2684 | 60.05119 |
| $P_{5}$ | 269.2216 | 71.84771 |
| $P_{6}$ | 106.9399 | 41.04252 |
| $P_{7}$ | 93.52947 | 38.24569 |
| $P_{8}$ | 291.7759 | 70.86596 |
| $S F 36_{\text {Total }}$ | 1666.325 | 267.0231 |

Distribution of each $P_{i}$ may be further transformed to common mean and SD say 50 and 10.

## Discussions:

The proposed method generating normally distributed scores for items, sub-classes and SF-36 scale contributes to improve scoring of the instrument and makes it possible to have meaningful single valued SF36 ${ }_{\text {Total }}$ scores satisfying desired properties.

## Benefits of the proposed methods are:

Better admissibility of arithmetic average, normally distributed scores
Parametric statistical analysis for meaningful comparisons over time and space, classification, testing of statistical hypothesis of equality of average of $S F 36_{\text {Total }}$ in two groups for cross-sectional as well as longitudinal data
Facilitates estimation of mean $(\mu)$, variance $\left(\sigma^{2}\right)$, confidence interval of $\mu$,Cronbach alpha at population level.
Responsiveness of SF-36 can be quantified reflecting ability of the scale to detect changes for an individual or for clinical samples and drawing of progress-paths. For non-clinical samples, changes in $S F 36_{\text {Total }}$ may be reflect effect of changes in major events of life.
Significance of responsiveness of SF-36 can be tested by $\chi^{2}$ test since ratio of two normally distributed variables follows $\chi^{2}$ distribution.
A better measure of validity of a multidimensional SF36 scale is proposed as the ratio of the first eigenvalue to the sum of all eigen values.

Discriminating value of Likert item ( Disc $_{i}$ ) and test discriminating value ( Disc $_{\text {Test }}$ ) were defined as CV and relationship derived between Cronbach $\alpha$ and Disc $_{\text {Test }}$ and Disc $_{\text {Test }}$ and theoretically defined $r_{t t}$.
Advantages of quartile clustering using normally distributed P -scores discussed. It is simple, appealing, adds clear meaning to the clusters, provides welldefined cut-off scores for the four mutually-exclusive classes and assigns equal probability to each quartile.
Researchers and practitioners in social and behavioral health can take advantages of the proposed method to find $S F 36_{\text {Total }}$ for meaningful comparison, assessment of progress or deterioration registered by patient(s) between successive time periods, avoiding limitations of summative Likert scores. Considering theoretical advantages, the proposed method of transforming raw scores of SF-36 items to normally distributed SF36 ${ }_{\text {Total }}$ scores is recommended for better inferences.

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