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Various Dominating sets of Sequential Join of Graphs

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Abstract

In this chapter, various types of dominating sets and their domination numbers, viz. domination number, total domination number, 2 - distance domination number, connected domination number and k - domination number of sequential join of graphs are determined. Moreover, we define another new operation named the alternating sequential join of two graphs and their domination and total domination numbers.

Keywords: Reference management software, Scopus, Bibliometric study, Citation.

Introduction

Sequential join is a graph operation, introduced by Buckley and Harary (1989). With the help of this operation, i.e. sequential join, many complex and big graphs can be designed, such as the construction of a communication network. This chapter contains 5 sections. Section 7.2 deals with domination number and total domination number of the join of any two graphs and the sequential join of any non-trivial connected graphs, respectively. In section 7.3, we derive some more results on domination number and total domination number of alternating sequential join of two graphs. Further, section 7.4 includes the 2 - distance domination number of the sequential join of any non-trivial connected graphs. The minimum connected domination number of the sequential join of graphs and its bound are calculated in Section 7.5. At last, we explore some observations on the minimum k - domination number of sequential join of path graphs, along with their upper and lower bounds of the k-domination number of sequential join of path graphs.

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Domination Number and Total Domination Number of Sequential Join of Graphs

In this section, we establish some standard results regarding the domination number and total domination number of general graphs by evaluating their minimum dominating set and minimum total dominating set.

Theorem 7.2.1

Let $G_1 + G_2$Be the join of two connected graphs $G_1$and G_2. Then:

$$\gamma(G_1 + G_2) = \begin{cases} 1, & \text{if } \gamma(G_1) = 1 \text{ or } \gamma(G_2) = 1, \\ 2, & \text{if } \gamma(G_1) \neq 1 \text{ and } \gamma(G_2) \neq 1. \end{cases}$$

Proof:

Suppose $\gamma(G_1) = 1$. Let $S = \{v\}$be a γ-set of G_1. Then $vx \in E(G_1 + G_2)$for each $x \in V(G_2)$By the definition of the join operation. Hence, SIs a dominating set in $G_1 + G_2$, and $\gamma(G_1 + G_2) = 1$. Similarly, if $\gamma(G_2) = 1$, then $\gamma(G_1 + G_2) = 1$.

Now, suppose $\gamma(G_1) \neq 1$and $\gamma(G_2) \neq 1$. Since every vertex of $G_1$is adjacent to all the vertices of G_2in $G_1 + G_2$, selecting one vertex from $G_1$and one vertex from G_2Yields a minimum dominating set. Therefore, $\gamma(G_1 + G_2) = 2$.

Corollary 7.2.1
If $G_1 + G_2$Is the join of two connected graphs, then:

$$\gamma(G_1 + G_2) = \begin{cases} 1, & \text{if } G_1 \text{ or } G_2 \text{ is complete,} \\ 2, & \text{if } G_1 \text{ or } G_2 \text{ is not complete.} \end{cases}$$

Theorem 7.2.2
Let $G_1 + G_2$Be the join of two connected graphs $G_1$and G_2. Then the total domination number is:

$$\gamma_t(G_1 + G_2) = 2.$$

Proof:
By definition, $\gamma(G) \leq \gamma_t(G)$. From Theorem 7.2.1, if $\gamma(G) = 2$The result follows directly. However, if $\gamma(G) = 1$the single-vertex dominating set cannot be a total dominating set because it contains an isolated vertex. A total dominating set (TDS) requires that every vertex in the set be adjacent to another vertex. Hence, $\gamma_t(G_1 + G_2) = 2$.

Theorem 7.2.3
Let G_mBe any graph in which no vertex is adjacent to all other vertices? Let $G = \Sigma_n G_m$Be the sequential join of $n$copies of G_m, with $m, n \geq 3$. Then the domination number and total domination number of GAs follows:

Proof:
Let $V = \{(i,j) \mid i = 1,2, \ldots, n; j = 1,2, \ldots, m\}$Be the vertex set of G. According to nWe have three cases:
Case 1: $n = 4k - 2, k \geq 1$

$$D = \{(4k_1 - 3,1), (4k_1 - 2,1) \mid 1 \leq k_1 \leq k\}. \text{es} \mid D \mid = 2k = 2\left\lceil \frac{n+2}{4} \right\rceil$$

Case 2: $n = 4k - 1$or $4k, k \geq 1$

$$D = \{(4k_1 - 2,1), (4k_1 - 1,1) \mid 1 \leq k_1 \leq k\}.$$

$$\text{Divides} \mid D \mid = 2k = 2\left\lceil \frac{n+2}{4} \right\rceil$$

In Cases 1 and 2, it can be verified that $D$is the minimum dominating set of $\Sigma_n(G_m)$. Obviously, no dominating set or total dominating set has cardinality smaller than $\mid D \mid$. Therefore:

$$\gamma(G) = \gamma_t(G) = \mid D \mid = 2\left\lceil \frac{n+2}{4} \right\rceil$$

Case 3: $n = 4k + 1, k \geq 1$

$$D = \{(2,1)\} \cup \{(4k_1 - 1,1), (4k_1, 1) \mid 1 \leq k_1 \leq k\}$$

$$\mid D \mid = 2k + 1 = \left\lceil \frac{n-1}{2} \right\rceil + 1 = n + 1$$

Thus, $D$is the minimum dominating set and minimum total dominating set for $\Sigma_n(G_m)$, so:

$$\gamma(G) = \gamma_t(G) = n + 1$$

Corollary 7.2.2
Let $G_m$be a path graph P_m, cycle graph C_m, or null graph $N_m$with $m > 3$. If $G = \Sigma_n G_m$with $n \geq 3$, then:

$$\gamma(G) = \gamma_t(G) = \begin{cases} 2\left\lceil \frac{n+2}{4} \right\rceil, & \text{if } n = 4k - 2, 4k - 1, 4k, \\ n + 1, & \text{if } n = 4k + 1, k \geq 1 \end{cases}$$

Example 7.2.1:
Let $G = \Sigma_8 P_5$. Then the sequential join of graph Gfollows the above rules for domination and total domination numbers.

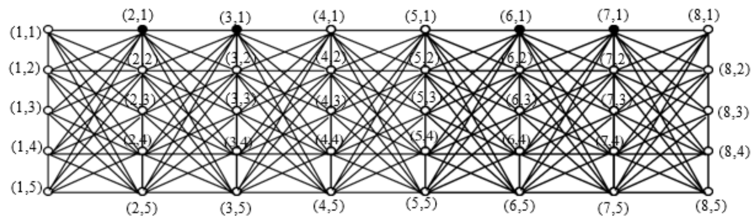

Figure 7.1: Sequential join of $\sum_{i=1}^{8} P_5$

In Figure 7.1, the sets {(2,1)}, {(3,1)}, {(6,1)}, and {(7,1)}are the minimum dominating sets and minimum total dominating sets of the graph.

Theorem 7.2.4
Let $G_m$be any graph in which at least one vertex is adjacent to all other vertices. If $G = \Sigma_n G_m$is the sequential join of $n$copies of $G_m$with $m, n \geq 3$, then the domination number of $G$is:

$$\gamma(G) = \left\lfloor \frac{n+2}{2} \right\rfloor.$$

Proof:
Let

$$V = \{(i,j) \mid i = 1,2, \dots, n;\ j = 1,2, \dots, m\}$$

be the vertex set of $\Sigma_n G_m$, where $n$denotes the number of copies and $m$denotes the number of vertices in each copy. According to the value of n, there are two cases:
Case 1: $n = 3k - 1$or $n = 3k$, $k \geq 1$

$$D = \{(3k_1 - 1,2) \mid 1 \leq k_1 \leq k\}.$$

Clearly, there is no dominating set with cardinality smaller than $\mid D \mid$. Thus, $D$is the minimum dominating set of $\Sigma_n G_m$, and

$$\gamma(G) = \mid D \mid = \left\lfloor \frac{n+2}{2} \right\rfloor.$$

Case 2: $n = 3k + 1$, $k \geq 1$

$$D = \{(3k_1 - 1,2), (3k_1 + 1,2) \mid 1 \leq k_1 \leq k\}.$$

Similarly, $\mid D \mid$is minimal, so $D$is the minimum dominating set of $\Sigma_n G_m$, and

$$\gamma(G) = \mid D \mid = \left\lfloor \frac{n+2}{2} \right\rfloor.$$

Theorem 7.2.5
Let $G_m$be any graph in which at least one vertex is adjacent to all other vertices. If $G = \Sigma_n G_m$is the sequential join of $n$copies of $G_m$with $m, n \geq 3$, then the total domination number of $G$is:

$$\gamma_t(G) = \begin{cases} 2\left\lfloor \frac{n+2}{2} \right\rfloor, & \text{if } n = 4k-2, 4k-1, 4k, \\ n+1, & \text{if } n = 4k+1,\ k \geq 1. \end{cases}$$

Proof:
Let

$$V = \{(i,j) \mid i = 1,2, \dots, n;\ j = 1,2, \dots, m\}$$

be the vertex set of $\Sigma_n G_m$. According to n, there are three cases:
Case 1: $n = 4k - 2, k \geq 1$

$$D = \{(4k_1 - 3,2), (4k_1 - 2,3) \mid 1 \leq k_1 \leq k\}.$$

Case 2: $n = 4k - 1$or $4k, k \geq 1$

$$D = \{(4k_1 - 2,2), (4k_1 - 1,3) \mid 1 \leq k_1 \leq k\}.$$

In Cases 1 and 2, there is no total dominating set with cardinality smaller than $| D |$, so D is the minimum total dominating set of $\Sigma_n G_m$. Hence,

$$\gamma_t(\Sigma_n G_m) = | D | = 2\left\lceil\frac{n+2}{2}\right\rceil.$$

Case 3: $n = 4k + 1, k \geq 1$

$$D = \{(2,2)\} \cup \{(4k_1 - 1,3), (4k_1, 2) \mid 1 \leq k_1 \leq k\}.$$

Clearly, $| D |$ is minimal, so D is the minimum total dominating set of $\Sigma_n G_m$, giving

$$\gamma_t(\Sigma_n G_m) = | D | = n + 1.$$

Alternating Sequential Join of Two Graphs

In this section, we establish results related to the domination number of alternating sequential join of two graphs, considering special classes of graphs such as P_m, C_m, N_m, K_m, W_m, and S_m. We also derive the minimum total dominating set of the alternating sequential join of any two general graphs.

Definition:

Let G_1 and G_2 be two graphs. The alternating sequential join of G_1 and G_2 is the join of a finite alternating sequence of graphs starting with G_1 and ending at either G_1 or G_2. Formally:

$$G_1 + G_2 + G_1 + G_1 + G_2 \text{ or } G_1 + G_2 + G_1 + G_1 + G_2 + G_1$$

The following figures show the graphical representation of both types of alternating sequential join of two graphs G_1 and G_2.

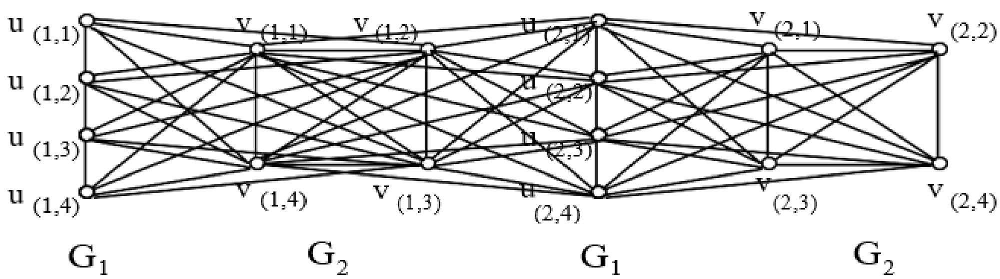

Figure 7.2: $G_1 + G_2 + G_1 + G_2$

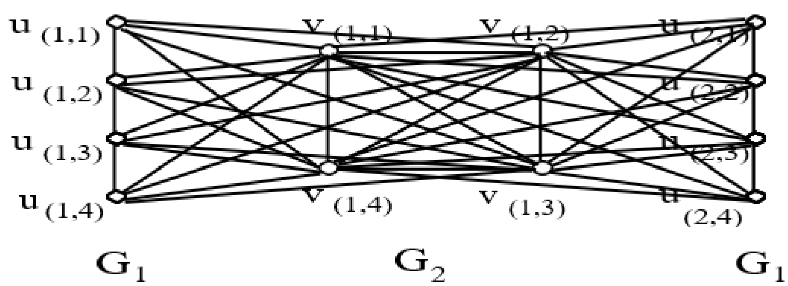

Figure 7.3: $G_1 + G_2 + G_1$

Theorem 7.3.1

Let G_1 and G_2 be any two graphs. Then the domination number of the alternating sequential join of n copies of G_1 and G_2 in total is as follows:

(i) If G_1 is either P_m, N_m, or C_m, and G_2 is fixed as K_m with $m > 3$, then

$$\gamma(G_1 + K_m + \cdots + G_1 + K_m) \text{ or } \gamma(G_1 + K_m + \cdots + G_1 + K_m + G_1) = j_n k$$

(ii) If G_1 is fixed as K_m, and G_2 is either P_m, N_m, or C_m, with $m > 3$, then

$$\gamma(K_m + G_2 + \cdots + K_m + G_2) \text{or} \gamma(K_m + G_2 + \cdots + K_m + G_2 + K_m) = l_n m$$

(iii) If G_1 is either P_m or N_m, and G_2 is fixed as C_m, $m > 3$, then

$$\gamma(G_1 + C_m + \cdots + G_1 + C_m) = \begin{cases} 2, & \text{if } n = 4k \\ n+1, & \text{if } n = 4k-2, k \in \mathbb{Z}, k \geq 1 \end{cases}$$

and

$$\gamma(G_1 + C_m + \cdots + G_1 + C_m + G_1) = n + 1$$

(iv) If G_1 is either K_m or W_m, and G_2 is fixed as S_m, $m > 3$, then

$$\gamma(G_1 + S_m + \cdots + G_1 + S_m) \text{or} \gamma(G_1 + S_m + \cdots + G_1 + S_m + G_1) = l_n m$$

Proof:
Let

$$V(G_1) = \{u_{i,j} \mid i = 1,2,3, \dots, n \ (n \geq 2), j = 1,2,3, \dots, m \ (m > 3)\}$$

and

$$V(G_2) = \{v_{i,j} \mid i = 1,2,3, \dots, n \ (n \geq 3), j = 1,2,3, \dots, m \ (m > 3)\}$$

be the vertex sets of $G_1 + G_2 + G_1 + \cdots + G_1 + G_2$, and

$$V(G_1) = \{u_{i,j} \mid i = 1,2,3, \dots, n+1 \ (n \geq 2), j = 1,2,3, \dots, m \ (m > 3)\},$$
$$V(G_2) = \{v_{i,j} \mid i = 1,2,3, \dots, n \ (n \geq 3), j = 1,2,3, \dots, m \ (m > 3)\}$$

be the vertex sets of $G_1 + G_2 + G_1 + \cdots + G_1 + G_2 + G_1$, where n denotes the number of copies of G_1 and G_2 in total, and m denotes the number of vertices in each copy.
According to the values of m and n, the minimum dominating sets for all sequential joins of the above graphs are determined in the following four cases:
Case (i): Consider G_1 as P_m, N_m, or C_m and G_2 as K_m, where $m > 3$. Assume D is the minimum dominating set of the alternating sequential join of the graphs:

(i) Consider G_1 as P_m, N_m, or C_m and G_2 as K_m, where $m > 3$. Let

$$D = \{v_n(k, 2) \mid 1 \leq k \leq j_n k\}.$$

It is clear that there is no dominating set with cardinality less than $\mid D \mid$ in the graph. Thus D is the minimum dominating set of the alternating sequential join of G_1 and K_m. Hence,

$$\gamma(G_1 + K_m + \cdots + G_1 + K_m) \text{or} \gamma(G_1 + K_m + \cdots + G_1 + K_m + G_1) = j_n k.$$

(ii) Consider G_1 as K_m and G_2 as P_m, N_m, or C_m, where $m > 3$. Let

$$D = \{u_n(k, 2) \mid 1 \leq k \leq l_n m\}.$$

Clearly, there is no dominating set with cardinality less than $\mid D \mid$ in the graph. Thus D is the minimum dominating set of the alternating sequential join of K_m and G_2. Hence,

$$\gamma(K_m + G_2 + \cdots + K_m + G_2) \text{or} \gamma(K_m + G_2 + \cdots + K_m + G_2 + K_m) = l_n m.$$

(iii) Consider G_1 as P_m or N_m and G_2 as C_m, where $m > 3$. Let D be the minimum dominating set of the alternating sequential join graphs. Depending on n, there are two cases:
Case 1: When $G_1 + C_m + G_1 + \cdots + G_1 + C_m$,

- (a) If $n = 4k (k \geq 1)$

$$D = \{v(2k_1 - 1,2), u(2k_1, 1) \mid 1 \leq k_1 \leq k\}.$$

- (b) If $n = 4k - 2 (k \geq 1)$

$$D = \{u(2k_1 - 1,1), v(2k_1 - 1,2) \mid 1 \leq k_1 \leq k\}.$$

Then,

$$\gamma(G_1 + C_m + \cdots + G_1 + C_m) = \begin{cases} 2, & \text{if } n = 4k \\ n+1, & \text{if } n = 4k-2, k \geq 1 \end{cases}$$

Case 2: When $G_1 + C_m + G_1 + \cdots + G_1 + C_m + G_1$,

- (a) If $n = 4k - 1 (k \geq 1)$
$$D = \{v(2k_1 - 1,2), u(2k_1, 1) \mid 1 \leq k_1 \leq k\}.$$

- (b) If $n = 4k + 1 (k \geq 1)$
$$D = \{v(2k_1 - 1,2), u(2k_1, 2) \mid 1 \leq k_1 \leq k\} \cup \{u(2k+1,1)\}.$$

Then,

$$\gamma(G_1 + C_m + \cdots + G_1 + C_m + G_1) = \frac{n+1}{2}.$$

(iv) Consider G_1 as K_m or W_m and G_2 as S_m, where $m > 3$. Let D be the minimum dominating set of the alternating sequential join graphs.

- (a) If $n = 3k + 1 (k \geq 1)$
$$D = \{v_n(2k - 1,1) \mid 1 \leq k \leq l_n m - j_n k\} \cup \{u_n(k, 1) \mid 1 \leq k \leq j_n k - j_n k\}.$$

- (b) If $n = 3k (k \geq 2)$ or $n = 3k + 2 (k \geq 1)$
$$D = \{v_n(3k - 2,1) \mid 1 \leq k \leq l_n m\} \cup \{u_n(3k, 1) \mid 1 \leq k \leq l_n m - 3\}.$$

Then,

$$\gamma(G_1 + S_m + \cdots + G_1 + S_m) \text{or} \gamma(G_1 + S_m + \cdots + G_1 + S_m + G_1) = l_n m.$$

Theorem 7.3.2

Let G_1 and G_2 be any two graphs. Then the total domination number of the alternating sequential join of the two graphs is:

$$\gamma_t(G_1 + G_2 + \cdots + G_1 + G_2) = \begin{cases} \frac{n+1}{2}, & \text{if } n = 4k, 4k-1, 4k-2, k \geq 1 \\ 4k-2, & \text{if } n = 4k+1, k \geq 1 \end{cases}$$

Proof: Let

$$V(G_1) = \{u_{i,j} \mid i = 1,2, \ldots, n\ (n \geq 2), j = 1,2, \ldots, m\ (m > 3)\},$$
$$V(G_2) = \{v_{i,j} \mid i = 1,2, \ldots, n\ (n \geq 3), j = 1,2, \ldots, m\ (m > 3)\}.$$

Assume D is the minimum total dominating set of the alternating sequential join graphs. Then:

- **Case 1:** When $n = 4k$, $4k - 1$, or $4k - 2$
$$D = \{v_n(2k_1 - 1,2) \mid 1 \leq k_1 \leq k\} \cup \{u_n(k_1 + 1,1) \mid 1 \leq k_1 \leq k\}.$$

- **Case 2:** When $n = 4k + 1$
$$D = \{v(1,2)\} \cup \{v_n(2k_1, 2) \mid 1 \leq k_1 \leq k\} \cup \{u_n(2k_1, 1) \mid 1 \leq k_1 \leq k\}.$$

It is clear that no total dominating set exists with cardinality less than $\mid D \mid$. Thus D is the minimum total dominating set of the alternating sequential join of G_1 and G_2.

Example 7.3.1

Let the graph $G = K_4 + S_4 + K_4 + S_4$. Then the alternating sequential join of graphs K_4 and S_4 is

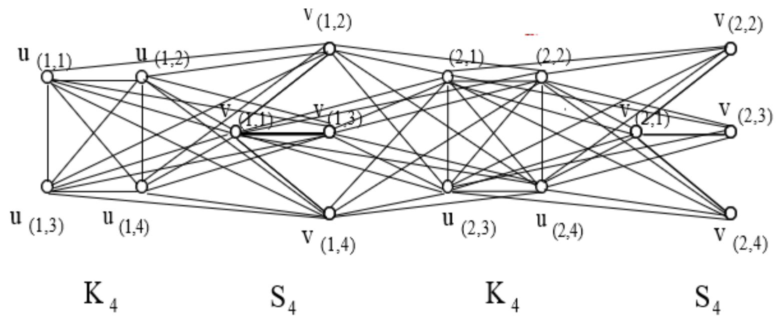

Figure 7.4: Alternating sequential join of $K_4 + S_4 + K_4 + S_4$

In **Figure 7.4**, the sets $\{v(1,1)\}$and $\{u(2,1)\}$are the **minimum dominating sets**, and $\{v(1,2)\}$and $\{u(2,1)\}$are the **minimum total dominating sets**.

7.4 2-Distance Domination Number of Sequential Join of Graphs

For any non-trivial connected graph, the **2-distance domination number** along with its **minimum 2-distance dominating set** of sequential join graphs are deduced in this section.

Theorem 7.4.1

Let Gbe a non-trivial connected graph of order n. Then the 2-domination number of the graph $\Sigma_n G$is:

$$\gamma_{\leq 2}(G) = \lceil n \rceil.$$

Proof:

Let

$$V = \{(i,j) \mid i = 1,2,3, \dots, n \ (n \geq 1), j = 1,2,3, \dots, m \ (m \geq 2)\}$$

be the vertex set of $\Sigma_n G_m$, where $n$denotes the number of copies and $m$denotes the number of vertices in each copy.

Case 1: If $n = 5k - 4$, $5k - 3$, or $5k - 2$, $k \geq 1$, let

$$D = \{(4k_1 - 2,1) \mid 1 \leq k_1 \leq k\}.$$

Clearly, there is no 2-distance dominating set with cardinality less than $\mid D \mid$. Thus $D$is the **minimum 2-distance dominating set** of $\Sigma_n(G)$. Hence,

$$\gamma_{\leq 2}[\Sigma_n(G)] = \mid D \mid = \lceil n \rceil.$$

Case 2: If $n = 5k - 1$or $5k$, $k \geq 1$, let

$$D = \{(5k_1 - 2,1) \mid 1 \leq k_1 \leq k\}.$$

Clearly, there is no 2-distance dominating set with cardinality less than $\mid D \mid$. Thus $D$is the **minimum 2-distance dominating set** of $\Sigma_n(G)$. Hence,

$$\gamma_{\leq 2}[\Sigma_n(G)] = \mid D \mid = \lceil n \rceil.$$

7.5 Connected Domination Number of Sequential Join of Graphs

In this section, we determine the **minimum connected domination number** along with the corresponding **minimum connected dominating set** of sequential joins of special classes of graphs.

Further, for non-trivial connected graphs, the **upper and lower bounds** with respect to $\gamma(G)$, $\gamma_t(G)$, and $\gamma_c(G)$are proposed. Some observations on **minimum k-domination numbers** of sequential joins of path graphs, along with their bounds, are also presented.

Theorem 7.5.1

If $G = \Sigma_n G_m$, where $G_m$is either $P_m$or $N_m$and $m, n \geq 4$, then the connected domination number of $G$is

$$\gamma_c(G) = n - 2.$$

Proof:

Let

$$V = \{(i,j) \mid i = 1,2,3, \dots, n \ (n > 3), j = 1,2,3, \dots, m \ (m > 3)\}.$$

Assume

$$D = \{(2,1), (k+2,1) \mid 1 \leq k \leq n-3\}.$$

Clearly, no connected dominating set has cardinality less than $\mid D \mid$. Thus $D$is the minimum connected dominating set of $\Sigma_n(G)$. Hence,

$$\gamma_c[\Sigma_n(G)] = \mid D \mid = n-2.$$

Theorem 7.5.2
If $G = \Sigma_n C_m$, where $m > 3$and $n \geq 3$, then

$$\gamma_c(G) = n-1.$$

Proof:
Let

$$V = \{(i,j) \mid i = 1,2,3,\ldots,n\ (n>3), j = 1,2,3,\ldots,m\ (m>3)\}.$$

Assume

$$D = \{(1,2), (k+1,2) \mid 1 \leq k \leq n-2\}.$$

Clearly, no connected dominating set has cardinality less than $\mid D \mid$. Thus $D$is the minimum connected dominating set of $\Sigma_n(G)$. Hence,

$$\gamma_c[\Sigma_n(G)] = \mid D \mid = n-1.$$

Theorem 7.5.3
If $G = \Sigma_n G_m$, where $G_m$is either K_m, W_m, or N_m, $m > 3$, and $n \geq 3$, then

$$\gamma_c(G) = n-2.$$

Proof:
Let

$$V = \{(i,j) \mid i = 1,2,3,\ldots,n\ (n>3), j = 1,2,3,\ldots,m\ (m>3)\}.$$

Assume

$$D = \{(k+1,2) \mid 1 \leq k \leq n-2\}.$$

Clearly, no connected dominating set has cardinality less than $\mid D \mid$. Thus $D$is the minimum connected dominating set of $\Sigma_n(G)$. Hence,

$$\gamma_c[\Sigma_n(G)] = \mid D \mid = n-2.$$

Observation 7.5.4
If $G = \Sigma_n G_m$, then

$$1 \leq \gamma(G) \leq \gamma_t(G) \leq \gamma_c(G) \leq n-1.$$

Observation 7.5.5
Let $G = \Sigma_n P_m$, with $m \geq k$and $n \geq 5$, $k, m \geq 2$. Then the minimum k-domination number of $G$is

$$\gamma_k(G) = \min\,(nm, k(n-2)).$$

Observation 7.5.6
If $G = \Sigma_n P_m$, with $m \leq k$and $n \geq 2$, $m \geq 2$, then the sharp bound of the minimum k-domination number of $G$is

$$m+n-1 \leq \gamma_k(\Sigma_n P_m) \leq n(k-2).$$

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