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Some Generalized Forms of Fuzzy Continuous Mappings

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ABSTRACT

Fuzzy topology, in particular, is a branch of the mathematical sciences that studies all problems pertaining to continuity, whether directly or indirectly. Fuzzy topology has seen the introduction and study of numerous continuous maps by topologists, all of which have been extremely beneficial to the advancement of modern science and technology. In this paper, we provide and examine a few generalized fuzzy continuous mappings and their interactions. Additionally, the findings have been corroborated with a few counterexamples.

Keywords: Generalized: fuzzy continuous mapping, fuzzy semi-continuous mapping, fuzzy pre-continuous mapping, fuzzy α-continuous mapping, fuzzy β-continuous mapping.

INTRODUCTION

Fuzzy set theory, first proposed by Lotfi A. Zadeh [19] in 1965, expands traditional set theory by introducing the concept of partial truth. In classical set theory, elements either fully belong to or are excluded from a set, but fuzzy set theory allows for varying degrees of membership, ranging between 0 and 1. This capability makes it especially useful for modeling uncertainty and imprecision in fields like control systems, decision-making under uncertain conditions, and pattern recognition. As a result, fuzzy set theory has proven more adaptable to solving practical, real-world problems compared to traditional set theory.

In 1968, C.L. Chang [06] contributed by establishing the concept of fuzzy topological spaces, and since then, many mathematicians have further developed the theory.

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For instance, Azad [01, 02] introduced the idea of fuzzy semi-open sets in fuzzy topological spaces. Bin Shahana [04, 05] followed with the introduction of fuzzy α-open and fuzzy pre-open sets. Thakur S.S. [15, 16, 17, 18] was the first to present the concept of fuzzy semi-pre-open sets in these spaces, while P. Chetty [10] discussed generalized fuzzy topology. Additionally, researchers such as Saraf R.K. and Thakur S.S. explored fuzzy virtually semi-continuous mappings and fuzzy pre-open mappings, contributing significant insights. Prasad R., Thakur S.S., and Saraf R.K. also examined fuzzy nearly α-continuous mappings and fuzzy irresolute α-mappings, verifying various results.

Continuous Mappings in Classical Mathematics

In the realm of classical mathematics, a mapping or function $f: \mathbb{X} \to \mathbb{Y}$ is considered continuous when small changes in the input $x \in \mathbb{X}$ correspond to small changes in the output $f(x) \in \mathbb{Y}$. normally, a function f is considered continuous if, for every fuzzy open set λ in the space $\mathbb{Y}$, the preimage $f^{-1}(\lambda)$ is an open set in $\mathbb{X}$.

Fuzzy Continuous Mappings

The concept of fuzzy continuous mappings generalizes the traditional notion of continuity to fuzzy contexts. In these scenarios, instead of handling precise sets and functions, we engage with fuzzy sets and fuzzy relations.

A mapping $f: \mathbb{X} \to \mathbb{Y}$ in a fuzzy context is considered fuzzy continuous if it preserves the fuzzy structure between the domain and codomain in a manner analogous to classical continuity.

Generalized Forms of Fuzzy Continuous Mappings

Generalized forms of fuzzy continuous mappings build upon the basic idea of fuzzy continuity by introducing various extensions and modifications to accommodate more complex scenarios. These generalizations can involve:

- Fuzzy Topological Spaces: These are extensions of classical topological spaces where open sets are replaced by fuzzy sets. Fuzzy continuous mappings in such spaces need to respect the fuzzy topological structure.
- α-Continuous Mappings: A mapping f is α-continuous if the preimage of an α -open set is also an α-open set. This concept generalizes the idea of continuity by considering different levels of openness.
- Fuzzy Uniform Spaces: These generalize uniform spaces to the fuzzy setting, allowing for the definition of uniform continuity and other related properties.
- Fuzzy Metric Spaces: These spaces extend classical metric spaces to fuzzy contexts, enabling the study of fuzzy continuous functions with respect to fuzzy metrics.

Applications and Importance

The study of generalized forms of fuzzy continuous mappings is crucial in various applications where uncertainty and vagueness are inherent. These include:

- Control Systems: Fuzzy controllers rely on fuzzy continuous mappings to model and manage imprecise inputs and outputs.
- Decision-Making: In scenarios involving uncertain or incomplete information, fuzzy continuous mappings help in formulating and solving decision problems.
- Pattern Recognition: Fuzzy continuous mappings facilitate the classification and analysis of patterns in data with inherent vagueness.

Understanding these generalized forms provides a deeper insight into the behavior of fuzzy systems and enhances their robustness and applicability in real-world problems.

02. Preliminaries

Definition 2.1 [10]: A family τ of fuzzy sets on $\mathbb{X}$ is called a generalized fuzzy topology on $\mathbb{X}$, if it satisfies the following conditions:

i) The null fuzzy set 0 and the whole fuzzy set 1 belongs to τ.

ii) Arbitrary union of members of τ is also in τ.

Then $(\mathbb{X},\tau)$ is called a generalized fuzzy topological space.

Definition 2.2 [.]: Let $(\mathbb{X},\tau)$ and $(\mathbb{Y},\tau)$ be two generalized fuzzy topological spaces on $\mathbb{X}$ and $\mathbb{Y}$. If $f:\mathbb{X}\to\mathbb{Y}$ be a mapping from $\mathbb{X}$ to $\mathbb{Y}$. Then f is called *generalized fuzzy continuous mapping* if $f^{-1}(\lambda)$ is generalized fuzzy open in $(\mathbb{X},\tau)$ for each fuzzy open set λ in $(\mathbb{Y},\tau)$.

Example 2.1 Let $\mathbb{X}=\{x_1,x_2\},\mathbb{Y}=\{y_1,y_2\}$ and X_0,X_1,X_2,X_3,X_4,X_5 belongs to $I^{\mathbb{X}}$ while as Y_0,Y_1,Y_2,Y_3,Y_4,Y_5 belongs to $I^{\mathbb{Y}}$ be fuzzy sets defined as:

$X_0=\{(x_1,0),(x_2,0)\}$, $X_1=\{(x_1,1),(x_2,1)\}$, $X_2=\{(x_1,0.5),(x_2,0.4)\}$,

$X_3=\{(x_1,0.4),(x_2,0.8)\}$, $X_4=\{(x_1,0.5),(x_2,0.7)\}$, $X_5=\{(x_1,0.5),(x_2,0.8)\}$.

And, $Y_0=\{(y_1,0),(y_2,0)\}$, $Y_1=\{(y_1,1),(y_2,1)\}$, $Y_2=\{(y_1,0.5),(y_2,0.4)\}$,

$Y_3=\{(y_1,0.4),(y_2,0.8)\}$, $Y_4=\{(y_1,0.5),(y_2,0.7)\}$, $Y_5=\{(y_1,0.5),(y_2,0.8)\}$.

Therefore, $(\mathbb{X},\tau)$ and $(\mathbb{Y},\tau)$ are generalized fuzzy topological spaces.

Let $f:\mathbb{X}\to\mathbb{Y}$ be a mapping from $\mathbb{X}$ to $\mathbb{Y}$ defined by

$f(X_i)=Y_j\ \ \forall\, i.j\in\{0,1,2,3,4.5\}$.

Then $f:\mathbb{X}\to\mathbb{Y}$ is generalized fuzzy continuous map.

Proposition 2.1: Let $(\mathbb{X},\tau)$ and $(\mathbb{Y},\tau)$ be generalized fuzzy topological spaces and $f:\mathbb{X}\to\mathbb{Y}$ be a mapping. Then the following statements are equivalent:

• f is generalized fuzzy continuous.

• $f^{-1}(\lambda)$ is generalized fuzzy closed in $(\mathbb{X},\tau)$ for each generalized fuzzy closed set λ in $(\mathbb{Y},\tau)$.

Proposition 2.2: Consider two generalized fuzzy topological spaces $(\mathbb{X},\tau)$ and $(\mathbb{Y},\tau)$, along with a mapping $f:\mathbb{X}\to\mathbb{Y}$. Suppose $\mathcal{B}$ is a basis for $(\mathbb{Y},\tau)$. If for each generalized fuzzy basic open set μ in $\mathcal{B}$, the preimage $f^{-1}(\mu)$ is generalized fuzzy open in $(\mathbb{X},\tau)$, then the mapping $f:\mathbb{X}\to\mathbb{Y}$ is generalized fuzzy continuous.

Proposition 2.3 Let $(\mathbb{X},\tau),(\mathbb{Y},\tau)$ and $(\mathbb{Z},\tau)$ be generalized fuzzy topological spaces. If $f:\mathbb{X}\to\mathbb{Y}$ and $g:\mathbb{Y}\to\mathbb{Z}$ are continuous mappings, then the composition $gof:\mathbb{X}\to\mathbb{Z}$ is also generalized fuzzy continuous mapping.

Remark 2.1: Let $(\mathbb{X},\tau),(\mathbb{Y},\tau)$ and $(\mathbb{Z},\tau)$ be generalized fuzzy topological spaces. If $f:\mathbb{Y}\times\mathbb{Z}\to\mathbb{Y}$ and $g:\mathbb{Y}\times\mathbb{Z}\to\mathbb{Z}$ are projection mappings, and $h:\mathbb{X}\to\mathbb{Y}\times\mathbb{Z}$ be generalized fuzzy continuous mapping, then both $foh:\mathbb{X}\to\mathbb{Y}$ and $goh:\mathbb{X}\to\mathbb{Z}$ are generalized fuzzy continuous mappings.

Proposition 2.4 Let $(\mathbb{X},\tau)$ and $(\mathbb{Y},\tau)$ be generalized fuzzy topological spaces. If $f:\mathbb{X}\to\mathbb{Y}$ be a generalized fuzzy continuous mapping and $g:\mathbb{X}\to\mathbb{X}\times\mathbb{Y}$ be the graph of function f, then g is generalized fuzzy continuous if and only if f is generalized fuzzy continuous mapping.

Proposition 2.5 Let $(\mathbb{X},\tau),(\mathbb{Y},\tau)$ and $(\mathbb{Z},\tau)$ be generalized fuzzy topological spaces. If $f:\mathbb{X}\to\mathbb{Z}$ and $g:\mathbb{Y}\to\mathbb{Z}$ are generalized fuzzy continuous mappings, then the mapping $f\times g:\mathbb{X}\times\mathbb{Y}\to\mathbb{Z}\times\mathbb{Z}$ is also generalized fuzzy continuous.

Definition 2.2 Let $(\mathbb{X},\tau)$ and $(\mathbb{Y},\tau)$ be generalized fuzzy topological spaces. If $f:\mathbb{X}\to\mathbb{Y}$ be a mapping, then f is called *generalized fuzzy* $\boldsymbol{\alpha}$*-continuous mapping* if $f^{-1}(\lambda)$ is generalized fuzzy α-open in $(\mathbb{X},\tau)$ for each generalized fuzzy open set λ in $(\mathbb{Y},\tau)$.

Example 2.2 Let $\mathbb{X}=\{x_1,x_2\},\mathbb{Y}=\{y_1,y_2\}$ and X_1,X_2,X_3,X_4 belongs to $I^{\mathbb{X}}$ while as Y_1,Y_2,Y_3,Y_4 belongs to $I^{\mathbb{Y}}$ be fuzzy sets defined as:

$X_1=\{(x_1,0.4),(x_2,0.7)\}$, $X_2=\{(x_1,0.5),(x_2,0.6)\}$,

$X_3=\{(x_1,0.5),(x_2,0.3)\}$, $X_4=\{(x_1,0.5),(x_2,0.7)\}$.

And, $Y_1=\{(y_1,0.3),(y_2,0.8)\}$, $Y_2=\{(y_1,0.4),(y_2,0.6)\}$,

$Y_3=\{(y_1,0.4),(y_2,0.3)\}$, $Y_4=\{(y_1,0.4),(y_2,0.8)\}$.

Therefore, $(\mathbb{X},\tau)$ and $(\mathbb{Y},\tau)$ are generalized fuzzy topological spaces.

Let $f:\mathbb{X}\to\mathbb{Y}$ be a mapping from $\mathbb{X}$ to $\mathbb{Y}$ defined by

$f(X_i)=Y_j\ \ \forall\, i,j\in\{1,2,3,4\}$.

Then $f: \mathbb{X} \to \mathbb{Y}$ is generalized fuzzy α-continuous map.

Remark 2.2: Every generalized fuzzy continuous mapping is generalized fuzzy α-continuous mapping and every fuzzy α-continuous map is fuzzy semi-continuous as well as fuzzy pre-continuous.

Proposition 2.6 Let $(\mathbb{X}, \tau)$ and $(\mathbb{Y}, \tau)$ be generalized fuzzy topological spaces, and let $f: \mathbb{X} \to \mathbb{Y}$ be a mapping. The following conditions are equivalent:

• f is generalized fuzzy α-continuous.

• For each generalized fuzzy closed λ in $(\mathbb{Y}, \tau)$, the preimage $f^{-1}(\lambda)$ is generalized fuzzy α-closed in $(\mathbb{X}, \tau)$.

• For each generalized fuzzy set μ in $(\mathbb{X}, \tau)$, α-closure of μ is contained in the closure of $f(\mu)$, i.e $\alpha - cl(\mu) \preccurlyeq cl(f(\mu))$.

• For each generalized fuzzy set λ in $(\mathbb{Y}, \tau)$, α-closure of $(f^{-1}(\lambda)$ is is contained in the preimage of the closure of λ, i.e. $\alpha - cl(f^{-1}(\lambda)) \preccurlyeq f^{-1}(cl(\lambda))$.

• For each generalized fuzzy set λ in $(\mathbb{Y}, \tau)$, the preimage of the interior of λ is is contained in the α-interior of the preimage of λ, i.e., $f^{-1}(int(\lambda)) \preccurlyeq \alpha - int(f^{-1}(\lambda))$.

Proposition 2.7 Let $(\mathbb{X}, \tau)$ and $(\mathbb{Y}, \tau)$ be generalized fuzzy topological spaces, and let $f: \mathbb{X} \to \mathbb{Y}$ be a mapping. If $\mathcal{B}$ be a basis for $(\mathbb{Y}, \tau)$, then f is *generalized fuzzy* $\boldsymbol{\alpha}$*-continuous mapping* if and only if $f^{-1}(\lambda)$ is generalized fuzzy α-open in $(\mathbb{X}, \tau)$ for each generalized fuzzy basic open set λ in $\mathcal{B}$.

Proposition 2.8 Let $(\mathbb{X}, \tau), (\mathbb{Y}, \tau)$ and $(\mathbb{Z}, \tau)$ be generalized fuzzy topological spaces, and let $f: \mathbb{X} \to \mathbb{Y}$ and $g: \mathbb{Y} \to \mathbb{Z}$ be the mappings. If such $f: \mathbb{X} \to \mathbb{Y}$ is generalized fuzzy α-continuous and $g: \mathbb{Y} \to \mathbb{Z}$ is generalized fuzzy continuous, then the composition $gof: \mathbb{X} \to \mathbb{Z}$ is also generalized fuzzy α-continuous mapping.

Proposition 2.9 Let $(\mathbb{X}, \tau), (\mathbb{Y}, \tau)$ and $(\mathbb{Z}, \tau)$ be generalized fuzzy topological spaces. Suppose $f: \mathbb{Y} \times \mathbb{Z} \to \mathbb{Y}$ and $g: \mathbb{Y} \times \mathbb{Z} \to \mathbb{Z}$ are projection mappings, and let $h: \mathbb{X} \to \mathbb{Y} \times \mathbb{Z}$ be a generalized fuzzy α-continuous mapping, then the compositions $foh: \mathbb{X} \to \mathbb{Y}$ and $goh: \mathbb{X} \to \mathbb{Z}$ are also generalized fuzzy α-continuous mappings.

Proposition 2.10 Let $(\mathbb{X}, \tau)$ and $(\mathbb{Y}, \tau)$ be generalized fuzzy topological spaces. If $f: \mathbb{X} \to \mathbb{Y}$ be a generalized fuzzy continuous mapping and $g: \mathbb{X} \to \mathbb{X} \times \mathbb{Y}$ be the graph of f, then f is generalized fuzzy α-continuous if g is generalized fuzzy α-continuous mapping.

Proposition 2.11 Let $(\mathbb{X}, \tau), (\mathbb{Y}, \tau)$ and $(\mathbb{Z}, \tau)$ be generalized fuzzy topological spaces. If $f: \mathbb{X} \to \mathbb{Z}$ and $g: \mathbb{Y} \to \mathbb{Z}$ are generalized fuzzy α-continuous mappings, then $f \times g: \mathbb{X} \times \mathbb{Y} \to \mathbb{Z} \times \mathbb{Z}$ is also generalized fuzzy α-continuous mapping.

Definition 2.3 Let $(\mathbb{X}, \tau)$ and $(\mathbb{Y}, \tau)$ be generalized fuzzy topological spaces, and let $f: \mathbb{X} \to \mathbb{Y}$ be a mapping, then f is called generalized fuzzy semi-continuous mapping if $f^{-1}(\lambda)$ is generalized fuzzy semi open in $(\mathbb{X}, \tau)$ for each generalized fuzzy open set λ in $(\mathbb{Y}, \tau)$.

Example 2.3 Let $\mathbb{X} = \{x_1, x_2\}, \mathbb{Y} = \{y_1, y_2\}$ and X_1, X_2, X_3, X_4 belongs to $I^{\mathbb{X}}$ while as Y_1, Y_2, Y_3, Y_4 belongs to $I^{\mathbb{Y}}$ be fuzzy sets defined as:

$X_1 = \{(x_1, 0.4), (x_2, 0.7)\}, X_2 = \{(x_1, 0.5), (x_2, 0.6)\}$,
$X_3 = \{(x_1, 0.5), (x_2, 0.3)\}, X_4 = \{(x_1, 0.5), (x_2, 0.7)\}$.
And, $Y_1 = \{(y_1, 0.3), (y_2, 0.8)\}, Y_2 = \{(y_1, 0.4), (y_2, 0.6)\}$,
$Y_3 = \{(y_1, 0.4), (y_2, 0.3)\}, Y_4 = \{(y_1, 0.4), (y_2, 0.8)\}$.

Therefore, $(\mathbb{X}, \tau)$ and $(\mathbb{Y}, \tau)$ are generalized fuzzy topological spaces.

Let $f: \mathbb{X} \to \mathbb{Y}$ be a mapping from $\mathbb{X}$ to $\mathbb{Y}$ defined by
$f(X_i) = Y_j \quad \forall\, i, j \in \{1, 2, 3, 4\}$.
Then $f: \mathbb{X} \to \mathbb{Y}$ is generalized fuzzy semi continuous map.

Remark 2.3: In a generalized fuzzy topological space every fuzzy continuous map is fuzzy semi-continuous but not converse.

Proposition 2.12 Let $(\mathbb{X}, \tau)$ and $(\mathbb{Y}, \tau)$ be generalized fuzzy topological spaces, and let $f: \mathbb{X} \to \mathbb{Y}$ be a mapping. The following conditions are equivalent:

• f is generalized fuzzy semi-continuous.

• For each generalized fuzzy closed set λ in $(\mathbb{Y}, \tau)$, the preimage $f^{-1}(\lambda)$ is a generalized fuzzy semi closed set in $(\mathbb{X}, \tau)$.

• For each generalized fuzzy set μ in $(\mathbb{X}, \tau)$, the image of interior of the closure of μ is contained in the closure of the image of μ, i.e. $f\left(int\left(cl(\mu)\right)\right) \preccurlyeq cl(f(\mu))$.

• For each generalized fuzzy set λ in $(\mathbb{Y}, \tau)$, the interior of closure if the preimage of λ is contained in the preimage closure of λ, i.e. $int(cl(f^{-1}(\lambda))) \preccurlyeq f^{-1}(cl(\lambda))$.

Proposition 2.13 Let $(\mathbb{X}, \tau), (\mathbb{Y}, \tau)$ and $(\mathbb{Z}, \tau)$ be generalized fuzzy topological spaces. Suppose $f: \mathbb{X} \to \mathbb{Y}$ is a generalized fuzzy semi-continuous mapping, and $g: \mathbb{Y} \to \mathbb{Z}$ is a generalized fuzzy continuous mapping. Then, the composition $gof: \mathbb{X} \to \mathbb{Z}$ is also a generalized fuzzy semi-continuous mapping.

Proposition 2.14 Consider generalized fuzzy topological spaces $(\mathbb{X}, \tau), (\mathbb{Y}, \tau)$ and $(\mathbb{Z}, \tau)$. Let $f: \mathbb{Y} \times \mathbb{Z} \to \mathbb{Y}$ and $g: \mathbb{Y} \times \mathbb{Z} \to \mathbb{Z}$ are the projection mappings. If $h: \mathbb{X} \to \mathbb{Y} \times \mathbb{Z}$ is a generalized fuzzy semi-continuous mapping, then the compositions $foh: \mathbb{X} \to \mathbb{Y}$ and $goh: \mathbb{X} \to \mathbb{Z}$ are also generalized fuzzy semi-continuous mappings.

Proposition 2.15 For the generalized fuzzy topological spaces $(\mathbb{X}, \tau)$ and $(\mathbb{Y}, \tau)$, let $f: \mathbb{X} \to \mathbb{Y}$ be a generalized fuzzy continuous mapping, and let $g: \mathbb{X} \to \mathbb{X} \times \mathbb{Y}$ be the graph of f. If g is generalized fuzzy semi-continuous mapping, then f itself is generalized fuzzy semi-continuous.

Definition 2.4 In the context of generalized fuzzy topological spaces $(\mathbb{X}, \tau)$ and $(\mathbb{Y}, \tau)$, a mapping $f: \mathbb{X} \to \mathbb{Y}$ is called *generalized fuzzy pre-continuous mapping* if for each generalized fuzzy open set λ in $(\mathbb{Y}, \tau)$, the inverse image $f^{-1}(\lambda)$ is generalized fuzzy pre-open in $(\mathbb{X}, \tau)$

Example 2.4 Let $\mathbb{X} = \{x_1, x_2\}, \mathbb{Y} = \{y_1, y_2\}$ and X_1, X_2, X_3, X_4 belongs to $I^{\mathbb{X}}$ while as Y_1, Y_2, Y_3, Y_4 belongs to $I^{\mathbb{Y}}$ be fuzzy sets defined as:

$X_1 = \{(x_1, 0.4), (x_2, 0.7)\}, X_2 = \{(x_1, 0.5), (x_2, 0.6)\}$,
$X_3 = \{(x_1, 0.5), (x_2, 0.3)\}, X_4 = \{(x_1, 0.5), (x_2, 0.7)\}$.
And, $Y_1 = \{(y_1, 0.3), (y_2, 0.8)\}, Y_2 = \{(y_1, 0.4), (y_2, 0.6)\}$,
$Y_3 = \{(y_1, 0.4), (y_2, 0.3)\}, Y_4 = \{(y_1, 0.4), (y_2, 0.8)\}$.

Therefore, $(\mathbb{X}, \tau)$ and $(\mathbb{Y}, \tau)$ are generalized fuzzy topological spaces.

Let $f: \mathbb{X} \to \mathbb{Y}$ be a mapping from $\mathbb{X}$ to $\mathbb{Y}$ defined by
$f(X_i) = Y_j \quad \forall\, i, j \in \{1, 2, 3, 4\}$.
Then $f: \mathbb{X} \to \mathbb{Y}$ is generalized fuzzy pre-continuous map.

Proposition 2.16 Let $(\mathbb{X},\tau)$ and $(\mathbb{Y},\tau)$ be generalized fuzzy topological spaces. For a mapping $f:\mathbb{X}\to\mathbb{Y}$, the following statements are equivalent:

• f is generalized fuzzy pre-continuous.

• For each generalized fuzzy closed set λ in $(\mathbb{Y},\tau)$, the inverse image $f^{-1}(\lambda)$ is generalized fuzzy pre-closed in $(\mathbb{X},\tau)$.

• For each generalized fuzzy set μ in $(\mathbb{X},\tau)$, $f\big(pcl(\mu)\big)\preccurlyeq cl\big(f(\mu)\big)$.

• For each generalized fuzzy set λ in $(\mathbb{Y},\tau)$, $pcl(f^{-1}(\lambda))\preccurlyeq f^{-1}(cl(\lambda))$.

• For each generalized fuzzy set λ in $(\mathbb{Y},\tau)$, $f^{-1}(int(\lambda))\preccurlyeq pint(f^{-1}(\lambda))$.

Proposition 2.17 Given three generalized fuzzy topological spaces $(\mathbb{X},\tau),(\mathbb{Y},\tau)$ and $(\mathbb{Z},\tau)$, if $f:\mathbb{X}\to\mathbb{Y}$ is a generalized fuzzy pre-continuous, and $g:\mathbb{Y}\to\mathbb{Z}$ is generalized fuzzy continuous, then the composition mapping $gof:\mathbb{X}\to\mathbb{Z}$ is also generalized fuzzy pre-continuous.

Proposition 2.18 Let $(\mathbb{X},\tau),(\mathbb{Y},\tau)$ and $(\mathbb{Z},\tau)$ be generalized fuzzy topological spaces. Consider the projections mappings $f:\mathbb{Y}\times\mathbb{Z}\to\mathbb{Y}$ and $g:\mathbb{Y}\times\mathbb{Z}\to\mathbb{Z}$. If $h:\mathbb{X}\to\mathbb{Y}\times\mathbb{Z}$ is generalized fuzzy pre-continuous mapping, then the composition mappings $foh:\mathbb{X}\to\mathbb{Y}$ and $goh:\mathbb{X}\to\mathbb{Z}$ are also generalized fuzzy pre-continuous.

Proposition 2.19 Let $(\mathbb{X},\tau)$ and $(\mathbb{Y},\tau)$ be generalized fuzzy topological spaces. If $f:\mathbb{X}\to\mathbb{Y}$ is a generalized fuzzy continuous mapping and $g:\mathbb{X}\to\mathbb{X}\times\mathbb{Y}$ represents the graph of f, then f is generalized fuzzy pre-continuous if g is generalized fuzzy pre-continuous mapping.

Proposition 2.20 For the generalized fuzzy topological spaces $(\mathbb{X},\tau),(\mathbb{Y},\tau)$ and $(\mathbb{Z},\tau)$, if $f:\mathbb{X}\to\mathbb{Z}$ and $g:\mathbb{Y}\to\mathbb{Z}$ are both generalized fuzzy pre-continuous mappings, then the product mapping $f\times g:\mathbb{X}\times\mathbb{Y}\to\mathbb{Z}\times\mathbb{Z}$ is also generalized fuzzy pre-continuous.

CONCLUSION

In this paper, we introduce and study some generalized fuzzy continuous mappings and study their relationship. Further the results has been verified by making some counter examples.

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