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Some Generalised Forms of Fuzzy Open Sets

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Abstract

In this manuscript, we have studied and introduced some generalized forms of fuzzy open sets and establish their relationships or implications on one another. Most of the earlier studies of open sets has been discussed in general topological spaces and fuzzy open sets in fuzzy topological spaces. Further most of the results which we have discussed are verified also by making use of some counter examples.

Keywords: Generalised: fuzzy open sets, fuzzy semi-open sets, fuzzy pre-open sets, fuzzy α-open sets, fuzzy β-open sets.

Introduction

Professor Lotfi A. Zadeh [01] in 1965 is the first who introduced the great concept of Fuzzy sets, provides a natural framework for generalizing many of the concepts of general topology to what might be called fuzzy topological spaces. The concept of fuzzy topological spaces was first introduced in 1968 by C.L. Chang [02]. Later on many mathematicians contributed in the development of the theory of fuzzy topological spaces. Azad [08-09] has introduced the concept of fuzzy semi-open sets in fuzzy topological spaces. Bin Shahana [10] has introduced the concepts of fuzzy pre-open sets and fuzzy α-opens in fuzzy topological space. Thakur S.S [06-07] has introduced the concept of fuzzy semi pre-open sets in fuzzy topological spaces. P. Chatty [04] is the first who discussed the generalized fuzzy topology. An alternative definition of fuzzy topology was given by Lowen [11]. Yalvac [12] introduced the concepts of fuzzy set and function on fuzzy spaces. In general topology, by introducing the notion of ideal [13], and several other authors carried out such analysis. There has been an extensive study on the importance of ideal in general topology in the paper of Janković & Hamlet [14]. Sarkar [15] introduced the notions of fuzzy ideal and fuzzy local function in fuzzy set theory. Mahmoud [16] investigated one application of fuzzy set theory. Hatir and Jafari [17] and Nasef and Hatir [18] defined fuzzy semi-*I*-open set and fuzzy pre-*I*-open set via fuzzy ideal. In the interest of brevity, we shall confine our attention in this note to the more concepts such as generalized fuzzy open sets in generalized fuzzy topological spaces. In section 2, some important definitions of generalized fuzzy topological spaces are given. In section 3, we have introduced some generalized fuzzy open sets in generalized fuzzy topological spaces, in which the results have been proved and verified by making the use of some counter examples

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2. Preliminaries

Definition 2.1: A family τ of fuzzy sets on $\mathbb{X}$ is called a generalized fuzzy topology on $\mathbb{X}$, if it satisfies the following conditions:

i) The null fuzzy set 0 and the whole fuzzy set 1 belongs to τ.
ii) Arbitrary union of members of τ is also in τ.
Then $(\mathbb{X}, \tau)$ is called a generalized fuzzy topological space.

Definition 2.2: Let $(\mathbb{X}, \tau)$ be a generalized fuzzy topological space, then the members of τ are called generalized fuzzy open sets, while the complement of the members of τ are called generalized fuzzy closed sets.

Remark 2.1: The collection $\{0, 1\}$ containing null fuzzy set 0 and the whole fuzzy set 1 is a fuzzy topology on $\mathbb{X}$ and may be called as generalized fuzzy indiscrete topology on $\mathbb{X}$. While as the collection containing all possible fuzzy sets on $\mathbb{X}$ is a generalized fuzzy topology on $\mathbb{X}$. This generalized fuzzy topology is called generalized fuzzy discrete topology on $\mathbb{X}$.

Proposition 2.1: Let $(\mathbb{X}, \tau)$ be a generalized fuzzy topological space then:

i) Arbitrary intersection of generalized fuzzy closed sets is a generalized fuzzy closed set.
ii) Finite union of generalized fuzzy closed sets is a generalized fuzzy closed set.

Definition 2.3: Let $(\mathbb{X}, \tau)$ be a generalized fuzzy topological space and let A be a fuzzy set on $\mathbb{X}$. Then the interior of fuzzy set A denoted by $int(A)$, is the largest open set contained in A.
Symbolically,
we write $int(A) = \{O : O \preccurlyeq A, O \in \tau\}$.
In other words, interior of a fuzzy set A in a generalized topological space $(\mathbb{X}, \tau)$ is the union of all generalized fuzzy open sets in $\mathbb{X}$ contained in A.

Definition 2.4: Let $(\mathbb{X}, \tau)$ be a generalized fuzzy topological space and let A be a fuzzy set on $\mathbb{X}$. Then the closer of fuzzy set A denoted by $cl(A)$, is the smallest closed superset of A.
Symbolically, we write $cl(A) = \{C : C \succcurlyeq A, C^c \in \tau\}$.
In other words, closer of a fuzzy set A in a generalized topological space $(\mathbb{X}, \tau)$ is the intersection of all generalized fuzzy closed sets in $\mathbb{X}$ containing A.

Proposition 2.1: Let A be a fuzzy set in a generalized fuzzy topological space $(\mathbb{X}, \tau)$ then

i) A is a generalized fuzzy open set iff $int(A) = A$.
ii) A is a generalized fuzzy closed set iff $cl(A) = A$.

Proposition 2.2: Let $(\mathbb{X}, \tau)$ be a generalized fuzzy topological space then for a fuzzy set A on $\mathbb{X}$

i) $1 - cl(A) = int(1 - A)$.
ii) $1 - int(A) = cl(1 - A)$.

Proposition 2.3: Let A and B be two fuzzy sets in a generalized fuzzy topological space $(\mathbb{X}, \tau)$. If $A \preccurlyeq B$, then

i) $int(A) \preccurlyeq int(B)$.
ii) $cl(A) \preccurlyeq cl(B)$.

Proposition 2.4: Let $\mathbb{X}$ be a universal set and let $\{\lambda_i\}_{i \in \wedge}$ be the family of generalized fuzzy sets on $\mathbb{X}$ for any index set $\wedge$. Then,

i) $\bigcup_{i\in\wedge} cl(\lambda_i) \preccurlyeq cl(\bigcup_{i\in\wedge} \lambda_i)$ for each index $i \in \wedge$.
ii) In case $\wedge$ is a finite set, $\bigcup_{i \in \wedge} cl(\lambda_i) \preccurlyeq cl(\bigcup_{i \in \wedge} \lambda_i)$.
iii) $\bigcup_{i \in \wedge} int(\lambda_i) \preccurlyeq int(\bigcup_{i \in \wedge} \lambda_i)$ for each index $i \in \wedge$.

3. Generalized forms of fuzzy open sets

Definition 3.1: If $(\mathbb{X}, \tau)$ is a generalized fuzzy topological space and A is any generalized fuzzy open set. Then A is said to be *generalized fuzzy semi-open* set if $A \preccurlyeq cl\big(int(A)\big)$.

Example 3.1: Let $\mathbb{X} = \{x_1, x_2\}$ and A, B, C and D belongs to $I^{\mathbb{X}}$ be fuzzy sets defined as
$A = \{(x_1, 0.4), (x_2, 0.7)\}$, $B = \{(x_1, 0.5), (x_2, 0.6)\}$,
$C = \{(x_1, 0.5), (x_2, 0.3)\}$, $D = \{(x_1, 0.5), (x_2, 0.7)\}$.
Therefore, $(\mathbb{X}, \tau)$ is generalized fuzzy topological space.
Clearly, $int(A) = A$ and $cl(A) = 1$.
Also, $cl\big(int(A)\big) = cl(A) = 1$ and $1 \succcurlyeq A$. This gives, $A \preccurlyeq cl\big(int(A)\big)$.
Therefore, A is generalized fuzzy semi-open set.

Proposition 3.1: Every generalized fuzzy open set is generalized fuzzy semi-open set.

Proof: Suppose $(\mathbb{X}, \tau)$ be a generalized fuzzy topological space and let A be any generalized fuzzy open set in $\mathbb{X}$. Then $int(A) = A$. Since $A \preccurlyeq cl(A)$. This gives, $cl(int(A) = cl(A) \succcurlyeq A$. Therefore, A is generalized fuzzy semi-open set. Hence the result.

Remark 3.1: The converse of Proposition 3.1 is not true in general.
In Example 3.1, Let $E = \{(x_1, 0.3), (x_2, 0.2)\}$ be any fuzzy set on $\mathbb{X}$. Clearly, $int(E) = 0$ and $cl(E) = E$.Also, $cl\big(int(E)\big) = cl(0) = \{(x_1, 0.5), (x_2, 0.3)\}and\{(x_1, 0.5), (x_2, 0.3)\} \succcurlyeq E$. This gives, $E \preccurlyeq cl\big(int(E)\big)$. Therefore, E is generalized fuzzy semi-open set, but E is not generalized fuzzy open set.

Definition 3.2: A fuzzy set A in a fuzzy topological space $(\mathbb{X}, \tau)$ is said to be *generalized fuzzy semi-closed* set if its complement $A^c = 1 - A$ is generalized fuzzy semi-open set in $\mathbb{X}$.

Remark 3.2: Every generalized fuzzy closed set in a generalized fuzzy topological space $(\mathbb{X},\tau)$ is generalized fuzzy semi-closed set but converse may not be true as shown below:

Example 3.2: In Example 3.1, Let $E=\{(x_1,0.3),(x_2,0.2)\}$ and $E^c=\{(x_1,0.7),(x_2,0.8)\}$ be any fuzzy sets on $\mathbb{X}$. Clearly $int(E)=0$ and $cl(E)=E$. Also, $cl\big(int(E)\big)=cl(0)=\{(x_1,0.5),(x_2,0.3)\}$ and $\{(x_1,0.5),(x_2,0.3)\}\succcurlyeq E$. This gives, $E\preccurlyeq cl\big(int(E)\big)$. Therefore, E is generalized fuzzy semi-open set, this implies E^c generalized fuzzy semi-closed set but E^c is not generalized fuzzy closed set.

Proposition 3.2: Show that if A and B are two generalized fuzzy semi-open sets then $A\cup B$ is also generalized fuzzy semi-open set.

Proof: Let A and B are any two generalized fuzzy semi-open sets.

Then $A\preccurlyeq cl\big(int(A)\big)$ and $B\preccurlyeq cl\big(int(B)\big)$.

We know that, $int(A)\cup int(B)\preccurlyeq int(A\cup B)$.

This gives, $cl[int(A)\cup int(B)]\preccurlyeq cl\big(int(A\cup B)\big)$.

This implies, $cl\big(int(A)\big)\cup cl\big(int(B)\big)\preccurlyeq cl\big(int(A\cup B)\big)$.

This further implies, $A\cup B\preccurlyeq cl\big(int(A\cup B)\big)$. ($\because$ A and B are semi-open)

Proposition 3.3: Show that arbitrary union of generalized fuzzy semi-open sets is generalized fuzzy semi-open set.

Proof: Let $(\mathbb{X},\tau)$ be a generalized fuzzy topological space and let $\{\lambda_i\}_{i\in\wedge}$ be a collection of generalized fuzzy semi-open sets in $\mathbb{X}$, where $\wedge$ is any index set.

Then, $\lambda_i\preccurlyeq cl\big(int(\lambda_i)\big)$ for every $i\in\wedge$.

We know that, $\bigcup_{i\in\wedge} int(\lambda_i)\preccurlyeq int(\bigcup_{i\in\wedge}\lambda_i)$.

This gives, $cl[\bigcup_{i\in\wedge} int(\lambda_i)]\preccurlyeq cl\big(int(\bigcup_{i\in\wedge}\lambda_i)\big)$.

This implies, $\bigcup_{i\in\wedge} cl\big(int(\lambda_i)\big)\preccurlyeq cl\big(int(\bigcup_{i\in\wedge}\lambda_i)\big)$.

This further implies, $\bigcup_{i\in\wedge}\lambda_i\preccurlyeq cl\big(int(\bigcup_{i\in\wedge}\lambda_i)\big)$. ($\because$ $\lambda_i\ \forall\ i\in\wedge$ are semi-open)

Proposition 3.4: Show that arbitrary intersection of generalized fuzzy semi-closed sets in $\mathbb{X}$ is a generalized fuzzy semi-closed set.

Proof: Let $(\mathbb{X},\tau)$ be a generalized fuzzy topological space and let $\{\lambda_i\}_{i\in\wedge}$ be a collection of generalized fuzzy semi-closed sets in $\mathbb{X}$, where $\wedge$ is any index set.

This implies, $\bigcup_{i\in\wedge}{\lambda_i}^c$ is a generalized fuzzy semi-open set in $\mathbb{X}$.

Therefore, $(\bigcup_{i\in\wedge}{\lambda_i}^c)^c=\bigcap_{i\in\wedge}\lambda_i$ is a generalized fuzzy semi-closed set in $\mathbb{X}$.

Hence, arbitrary intersection of generalized fuzzy semi-closed sets is a generalized fuzzy semi-closed set.

Definition 3.3: If $(\mathbb{X},\tau)$ is a generalized fuzzy topological space and A is any generalized fuzzy open set. Then A is said to be *generalized fuzzy pre-open set* if $A\preccurlyeq int\big(cl(A)\big)$.

By Example 3.2, Clearly, $int(A)=A$ and $cl(A)=1$.

Also, $int\big(cl(A)\big)=int(1)=\{(x_1,0.5),(x_2,0.7)\}$ and $\{(x_1,0.5),(x_2,0.7)\}\succcurlyeq A$.

This gives, $A\preccurlyeq int\big(cl(A)\big)$.

Proposition 3.5: Every generalized fuzzy open set is generalized fuzzy pre-open set.

Proof Suppose $(\mathbb{X},\tau)$be a generalized fuzzy topological space and let A be any generalized fuzzy open set in$\mathbb{X}$. Then, $int(A)=A$. Since $A\preccurlyeq cl(A)$. This gives, $int(A)\preccurlyeq int(cl(A))$.

This implies, $A\preccurlyeq int(cl(A))$. Therefore, A is generalized fuzzy pre-open set.

Remark 3.3: The converse of Proposition 3.5 is not true in general.

In Example 3.2, Let $E=\{(x_1,0.3),(x_2,0.2)\}$ be any fuzzy set on $\mathbb{X}$. Clearly, $int(E)=0$ and $cl(E)=\{(x_1,0.5),(x_2,0.3)\}$.Also,$int\big(cl(E)\big)=int(\{(x_1,0.5),(x_2,0.3)\})=\{(x_1,0.5),(x_2,0.3)\}$. And $\{(x_1,0.5),(x_2,0.3)\}\succcurlyeq E$. This gives, $E\preccurlyeq int\big(cl(E)\big)$. Therefore, E is generalized fuzzy pre-open set, but E is not generalized fuzzy open set.

Definition 3.4: A fuzzy set A in a fuzzy topological space $(\mathbb{X},\tau)$is said to be *generalized fuzzy pre-closed set* if its complement $A^c=1-A$ is generalized fuzzy pre-open set in$\mathbb{X}$.

Remark 3.4: Every generalized fuzzy closed set in a generalized fuzzy topological space $(\mathbb{X},\tau)$ is generalized fuzzy pre-closed set but converse may not be true as shown below:

In Example 3.2, Let, $E=\{(x_1,0.3),(x_2,0.2)\}$ and $E^c=\{(x_1,0.7),(x_2,0.8)\}$ be any fuzzy sets on $\mathbb{X}$. Clearly, $int(E)=0$ and $cl(E)=E$.Also, $int\big(cl(E)\big)=int(E)=\{(x_1,0.5),(x_2,0.3)\}$ and $\{(x_1,0.5),(x_2,0.3)\}\succcurlyeq E$. This gives, $E\preccurlyeq int\big(cl(E)\big)$. Therefore, E is generalized fuzzy pre-open set, this implies E^c is generalized fuzzy pre-closed set but E^c is not generalized fuzzy closed set.

Proposition 3.6: Show that if A and B are two generalized fuzzy pre-open sets then $A\cup B$ is also generalized fuzzy pre-open set.

Proof If A and B are two generalized fuzzy pre-open sets.

Then, $A\preccurlyeq int\big(cl(A)\big)$ and $B\preccurlyeq int\big(cl(B)\big)$.

We know that, $cl(A)\cup cl(B)\preccurlyeq cl(A\cup B)$.

This gives, $int[cl(A)\cup cl(B)]\preccurlyeq int\big(cl(A\cup B)\big)$

$\Rightarrow int\big(cl(A)\big)\cup int\big(cl(B)\big)\preccurlyeq int\big(cl(A\cup B)\big)$.

$\Rightarrow A \cup B \preccurlyeq int\big(cl(A \cup B)\big)$.

($\because$ A and B $are\ pre - open$)

Therefore, union of two generalized fuzzy pre-open sets is also generalized fuzzy pre-open set. Hence the, result.

Proposition 3.7: Show that arbitrary union of generalized fuzzy pre-open sets is generalized fuzzy pre-open set.

Proof Let $(\mathbb{X}, \tau)$ be a generalized fuzzy topological space and let $\{\lambda_i\}_{i\in\wedge}$ be a collection of generalized fuzzy pre-open sets in $\mathbb{X}$, where $\wedge$ is any index set.

Then, $\lambda_i \preccurlyeq int\big(cl(\lambda_i)\big)$ for every $i \in \wedge$.

We know that, $\bigcup_{i\in\wedge} cl(\lambda_i) \preccurlyeq cl(\bigcup_{i\in\wedge} \lambda_i)$.

This gives, $int[\bigcup_{i\in\wedge} cl(\lambda_i)] \preccurlyeq int\big(cl(\bigcup_{i\in\wedge} \lambda_i)\big)$.

$\Rightarrow \bigcup_{i\in\wedge} int(cl(\lambda_i)) \preccurlyeq int(cl(\bigcup_{i\in\wedge} \lambda_i)$.

$\Rightarrow \bigcup_{i\in\wedge} \lambda_i \preccurlyeq int(cl(\bigcup_{i\in\wedge} \lambda_i))$ ($\because$ $\lambda_i\ for\ each\ i \in \wedge\ are\ semiopen$)

Therefore, arbitrary union of generalized fuzzy pre-open sets is also generalized fuzzy pre-open set. Hence, the result.

Proposition 3.8: show that arbitrary intersection of generalized fuzzy pre-closed sets in $\mathbb{X}$ is a generalized fuzzy pre-closed set.

Proof Let $(\mathbb{X}, \tau)$ be a generalized fuzzy topological space and let $\{\lambda_i\}_{i\in\wedge}$ be a collection of generalized fuzzy pre-closed sets in $\mathbb{X}$, where $\wedge$ is any index set.

This implies $\bigcup_{i\in\wedge} {\lambda_i}^c$ is a generalized fuzzy pre-open set in $\mathbb{X}$.

Therefore, $(\bigcup_{i\in\wedge} {\lambda_i}^c)^c = \bigcap_{i\in\wedge} \lambda_i$ is a generalized fuzzy pre-closed set in $\mathbb{X}$.

Hence, arbitrary intersection of generalized fuzzy pre-closed sets is a generalized fuzzy pre-closed set.

Definition 3.5: If $(\mathbb{X}, \tau)$ is a generalized fuzzy topological space and A is any generalized fuzzy open set. Then A is said to be *generalized fuzzy α-open set* if $A \preccurlyeq int\Big(cl\big(int(A)\big)\Big)$

By Example 3.2, Clearly, $int(A) = A$ and $cl(int(A)) = cl(A) = 1$.

Also, $int\Big(cl\big(int(A)\big)\Big) = int(1) = \{(x_1, 0.5), (x_2, 0.7)\}$ and $\{(x_1, 0.5), (x_2, 0.7)\} \succcurlyeq A$.

This gives, $A \preccurlyeq int\Big(cl\big(int(A)\big)\Big)$. Therefore, A is generalized fuzzy $\underline{\alpha}$-open set.

Proposition 3.9: Every generalized fuzzy open set is generalized fuzzy α-open set.

Proof: Suppose $(\mathbb{X}, \tau)$be a generalized fuzzy topological space and let A be any generalized fuzzy open set in$\mathbb{X}$. Then, $int(A) = A$. Since $A \preccurlyeq cl(A)$.

This gives, $cl(int(A) = cl(A) \succcurlyeq A$. This further implies, $int(cl\big(int(A)\big) \succcurlyeq int(A) = A$.

Therefore, A is generalized fuzzy α-open set.

Remark 3.5: But converse of the Proposition 3.9 may not be true in general.

In Example 3.2, Let, $E = \{(x_1, 0.3), (x_2, 0.2)\}$ be any fuzzy set on $\mathbb{X}$.

Clearly, $int(E) = 0$ and $cl(intE)) = cl(0) = \{(x_1, 0.5), (x_2, 0.3)\}$.

Also, $int\Big(cl\big(int(E)\big)\Big) = int(\{(x_1, 0.5), (x_2, 0.3)\}) = \{(x_1, 0.5), (x_2, 0.3)\}$.

and $\{(x_1, 0.5), (x_2, 0.3)\} \succcurlyeq E$. This gives, $E \preccurlyeq int\Big(cl\big(int(E)\big)\Big)$.

Therefore, E is generalized fuzzy α-open set, but E is not generalized fuzzy open set.

Definition 3.6: A fuzzy set A in a fuzzy topological space $(\mathbb{X}, \tau)$is said to be *generalized fuzzy α-closed set* if its complement $A^c = 1 - A$ is generalized fuzzy α-open set in $\mathbb{X}$.

Remark 3.6: Every generalized fuzzy closed set in a generalized fuzzy topological space $(\mathbb{X}, \tau)$ is generalized fuzzy α-closed set but converse may not be true as shown below:

In Example 3.2, Let, $E = \{(x_1, 0.3), (x_2, 0.2)\}$ and $E^c = \{(x_1, 0.7), (x_2, 0.8)\}$ be any fuzzy sets on $\mathbb{X}$. Clearly, $int(E) = 0$ and $cl(int(E)) = cl(0) = \{(x_1, 0.5), (x_2, 0.3)\}$.

Also, $int\Big(cl\big(int(E)\big)\Big) = int(\{(x_1, 0.5), (x_2, 0.3)\}) = \{(x_1, 0.5), (x_2, 0.3)\}$.

And $\{(x_1, 0.5), (x_2, 0.3)\} \succcurlyeq E$. This gives, $E \preccurlyeq int\Big(cl\big(int(E)\big)\Big)$.

Therefore, E is generalized fuzzy α-open set, this implies E^cgeneralized fuzzy α-closed set but E^c is not generalized fuzzy closed set.

Proposition 3.10: Show that if A and B are two generalized fuzzy α-open sets then $A{\cup}B$ is also generalized fuzzy α-open set.

Proof: If A and B are two generalized fuzzy α-open sets.

Then, $A \preccurlyeq int\Big(cl\big(int(A)\big)\Big)$ and $B \preccurlyeq int\Big(cl\big(int(A)\big)\Big)$.

We know that, $int(A) \cup int(B) \preccurlyeq int(A \cup B)$.

This gives, $cl[int(A) \cup int(B)] \preccurlyeq cl\big(int(A \cup B)\big)$.

$\Rightarrow cl\big(int(A)\big) \cup cl\big(int(B)\big) \preccurlyeq cl\big(int(A \cup B)\big)$.

This gives, $int\big[cl\big(int(A)\big) \cup cl\big(int(B)\big)\big] \preccurlyeq int\Big(cl\big(int(A \cup B)\big)\Big)$.

This gives, $int\Big(cl\big(int(A)\big)\Big) \cup int\Big(cl\big(int(B)\big)\Big) \preccurlyeq int\Big(cl\big(int(A \cup B)\big)\Big)$.

$\Rightarrow A \cup B \preccurlyeq int\Big(cl\big(int(A \cup B)\big)\Big)$.

($\because$ A and B $are\ \alpha - open$)

Therefore, union of two generalized fuzzy α-open sets is also generalized fuzzy α-open set.

Proposition 3.11: Show that arbitrary union of generalized fuzzy α-open sets is generalized fuzzy α-open set.

Proof: Let $(\mathbb{X}, \tau)$ be a generalized fuzzy topological space and let $\{\lambda_i\}_{i\in\wedge}$ be a collection of generalized fuzzy α-open sets in $\mathbb{X}$, where $\wedge$ is any index set.
Then, $\lambda_i \preccurlyeq int(cl(int(\lambda_i)))$ for every $i \in \wedge$.
We know that, $\cup_{i\in\wedge}(int(\lambda_i)) \preccurlyeq int(\cup_{i\in\wedge} \lambda_i)$.
This gives, $cl[\cup_{i\in\wedge} int(\lambda_i)] \preccurlyeq cl(int(\cup_{i\in\wedge} \lambda_i))$.
$\Rightarrow \cup_{i\in\wedge} cl(int(\lambda_i)) \preccurlyeq cl(int(\cup_{i\in\wedge} \lambda_i))$.
This gives, $int(\cup_{i\in\wedge} cl(int(\lambda_i))) \preccurlyeq int\left(cl(int(\cup_{i\in\wedge} \lambda_i))\right)$.
$\Rightarrow \cup_{i\in\wedge} cl(int(\lambda_i)) \preccurlyeq int\left(cl(int(\cup_{i\in\wedge} \lambda_i))\right)$.
$\Rightarrow \cup_{i\in\wedge} \lambda_i \preccurlyeq int\left(cl(int(\cup_{i\in\wedge} \lambda_i))\right)$.
$(\because \lambda_i \; for \; each \; i \in \wedge \; are \; semiopen)$
Therefore, arbitrary union of generalized fuzzy α-open sets is also generalized fuzzy α-open set. Hence, the result.

Proposition 3.12: Show that arbitrary intersection of generalized fuzzy α-closed sets in $\mathbb{X}$ is a generalized fuzzy α-closed set.

Proof: Let $(\mathbb{X}, \tau)$ be a generalized fuzzy topological space and let $\{\lambda_i\}_{i\in\wedge}$ be a collection of generalized fuzzy α-closed sets in $\mathbb{X}$, where $\wedge$ is any index set.
This implies $\cup_{i\in\wedge} {\lambda_i}^c$ is a generalized fuzzy α-open set in $\mathbb{X}$.
Therefore, $(\cup_{i\in\wedge} {\lambda_i}^c)^c = \cap_{i\in\wedge} \lambda_i$ is a generalized fuzzy α-closed set in $\mathbb{X}$.
Hence, arbitrary intersection of generalized fuzzy α-closed sets is a generalized fuzzy α-closed set.

Definition 3.7: If $(\mathbb{X}, \tau)$ is a generalized fuzzy topological space and A is any generalized fuzzy open set. Then A is said to be *generalized fuzzy β-open set* if $A \preccurlyeq cl\left(int(cl(A))\right)$.
By Example 3.2, Clearly, $cl(A) = \{(x_1, 0.5), (x_2, 0.7)\}$ and $int(cl(A)) = int(\{(x_1, 0.5), (x_2, 0.7)\}) = D$.
Also, $cl\left(int(cl(A))\right) = cl(D) = \{(x_1, 0.5), (x_2, 0.7)\}$ and $\{(x_1, 0.5), (x_2, 0.7)\} \succcurlyeq A$.
This gives, $A \preccurlyeq cl\left(int(cl(A))\right)$. Therefore, A is generalized fuzzy β-open set.

Remark 3.7: Fuzzy β-open set is also called fuzzy *semi pre-open set*.

Proposition 3.13: Every generalized fuzzy open set is generalized fuzzy β-open set.

Proof: Suppose $(\mathbb{X}, \tau)$be a generalized fuzzy topological space and let A be any generalized fuzzy open set in$\mathbb{X}$. Then, $int(A) = A$. Since $A \preccurlyeq cl(A)$.
This gives, $A = int(A) \preccurlyeq int(cl(A))$. This further implies, $cl(A) \preccurlyeq cl\left(int(cl(A))\right)$. This gives, $A \preccurlyeq cl(A) \preccurlyeq cl\left(int(cl(A))\right)$. Therefore, A is generalized fuzzy β-open set.

Remark 3.8: But converse of proposition 3.13 may not be true in general.
In Example 3.2, Let, $E = \{(x_1, 0.3), (x_2, 0.2)\}$ be any fuzzy set on $\mathbb{X}$.
Clearly, $cl(E) = \{(x_1, 0.5), (x_2, 0.3)\}$
and $int(cl(E)) = cl(\{(x_1, 0.5), (x_2, 0.3)\}) = \{(x_1, 0.5), (x_2, 0.3)\}$.
Also, $cl\left(int(cl(E))\right) = cl(\{(x_1, 0.5), (x_2, 0.3)\}) = \{(x_1, 0.5), (x_2, 0.3)\}$.
and $\{(x_1, 0.5), (x_2, 0.3)\} \succcurlyeq E$. This gives, $E \preccurlyeq cl\left(int(cl(E))\right)$. Therefore, E is generalized fuzzy β-open set, but E is not generalized fuzzy open set.

Definition 3.8: A fuzzy set A in a fuzzy topological space $(\mathbb{X}, \tau)$is said to be *generalized fuzzy β-closed set* if its complement $A^c = 1 - A$ is generalized fuzzy β-open set in$\mathbb{X}$.

Remark 3.9: Every generalized fuzzy closed set in a generalized fuzzy topological space $(\mathbb{X}, \tau)$ is generalized fuzzy β-closed set but converse may not be true as shown below:
In Example 3.2, Clearly, $cl(E) = \{(x_1, 0.5), (x_2, 0.3)\}$
and $int(cl(E)) = cl(\{(x_1, 0.5), (x_2, 0.3)\}) = \{(x_1, 0.5), (x_2, 0.3)\}$.
Also, $cl\left(int(cl(E))\right) = cl(\{(x_1, 0.5), (x_2, 0.3)\}) = \{(x_1, 0.5), (x_2, 0.3)\}$
and $\{(x_1, 0.5), (x_2, 0.3)\} \succcurlyeq E$. This gives, $E \preccurlyeq cl\left(int(cl(E))\right)$.
Therefore, E is generalized fuzzy β-open set, this implies E^cgeneralized fuzzy β-closed set but E^c is not generalized fuzzy closed set.

Proposition 3.14: Show that if A and B are two generalized fuzzy β-open sets then $A \cup B$ is also generalized fuzzy β-open set.

Proof: If A and B are two generalized fuzzy β-open sets.
Then, $A \preccurlyeq cl\left(int(cl(A))\right)$ and $B \preccurlyeq cl\left(int(cl(A))\right)$.
We know that, $cl(A) \cup cl(B) \preccurlyeq cl(A \cup B)$.
This gives, $int[cl(A) \cup cl(B)] \preccurlyeq int(cl(A \cup B))$.
$\Rightarrow int(cl(A)) \cup int(cl(B)) \preccurlyeq int(cl(A \cup B))$.
This gives, $cl[int(cl(A)) \cup int(cl(B))] \preccurlyeq cl\left(int(cl(A \cup B))\right)$.
This gives, $cl\left(int(cl(A))\right) \cup cl\left(int(cl(B))\right) \preccurlyeq cl\left(int(cl(A \cup B))\right)$.

$\Rightarrow A \cup B \preccurlyeq cl\left(int\big(cl(A \cup B)\big)\right)$.

$(\because A$ and $B\ are\ \beta - open)$

Therefore, union of two generalized fuzzy β-open sets is also generalized fuzzy β-open set.

Proposition 3.15: Show that arbitrary union of generalized fuzzy β-open sets is generalized fuzzy β-open set.

Proof: Let $(\mathbb{X}, \tau)$ be a generalized fuzzy topological space and let $\{\lambda_i\}_{i\in\wedge}$ be a collection of generalized fuzzy β-open sets in $\mathbb{X}$, where $\wedge$ is any index set.
Then, $\lambda_i \preccurlyeq cl\big(int\big(cl(\lambda_i)\big)\big)$ for every $i \in \wedge$.
We know that, $\bigcup_{i\in\wedge}\big(cl(\lambda_i)\big) \preccurlyeq cl(\bigcup_{i\in\wedge}\lambda_i)$.
This gives, $int[\bigcup_{i\in\wedge} cl(\lambda_i)] \preccurlyeq int\big(cl(\bigcup_{i\in\wedge}\lambda_i)\big)$.
This implies, $\bigcup_{i\in\wedge} int\big(cl(\lambda_i)\big) \preccurlyeq int\big(cl(\bigcup_{i\in\wedge}\lambda_i)\big)$.
Further, $cl\big(\bigcup_{i\in\wedge} int\big(cl(\lambda_i)\big)\big) \preccurlyeq cl\left(int\big(cl(\bigcup_{i\in\wedge}\lambda_i)\big)\right)$.
This gives, $\bigcup_{i\in\wedge} int\big(cl(\lambda_i)\big) \preccurlyeq cl\left(int\big(cl(\bigcup_{i\in\wedge}\lambda_i)\big)\right)$.
$\Rightarrow \bigcup_{i\in\wedge}\lambda_i \preccurlyeq cl\left(int\big(cl(\bigcup_{i\in\wedge}\lambda_i)\big)\right)$. $(\because$ λ_i for each $i \in \wedge$ are pre $-$ open)

Therefore, arbitrary union of generalized fuzzy β-open sets is also generalized fuzzy β-open set. Hence, the result.

Proposition 3.16: Show that arbitrary intersection of generalized fuzzy β-closed sets in $\mathbb{X}$ is a generalized fuzzy β-closed set.

Proof: Let $(\mathbb{X}, \tau)$ be a generalized fuzzy topological space and let $\{\lambda_i\}_{i\in\wedge}$ be a collection of generalized fuzzy β-closed sets in $\mathbb{X}$, where $\wedge$ is any index set.
This implies $\bigcup_{i\in\wedge}{\lambda_i}^c$ is a generalized fuzzy β-open set in $\mathbb{X}$.
Therefore, $(\bigcup_{i\in\wedge}{\lambda_i}^c)^c = \bigcap_{i\in\wedge}\lambda_i$ is a generalized fuzzy β-closed set in $\mathbb{X}$.
Hence, arbitrary intersection of generalized fuzzy β-closed sets is a generalized fuzzy β-closed set.

Conclusion Remark: We conclude the main results of this paper in the form of Figure 1 given below in which GF implies Generalized Fuzzy:

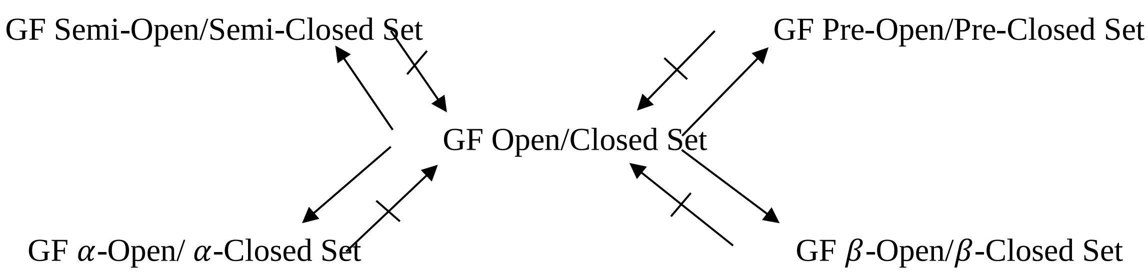

Figure 1

CONCLUSIONS

The results presented in this paper indicates that many of the basic concepts in general topology or fuzzy topological spaces can readily be extended to generalized fuzzy topological spaces. Although the theory of fuzzy sets cum fuzzy open sets is still evolving and yet to reach its full potential, it shows promise of having wide research area and applications.

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